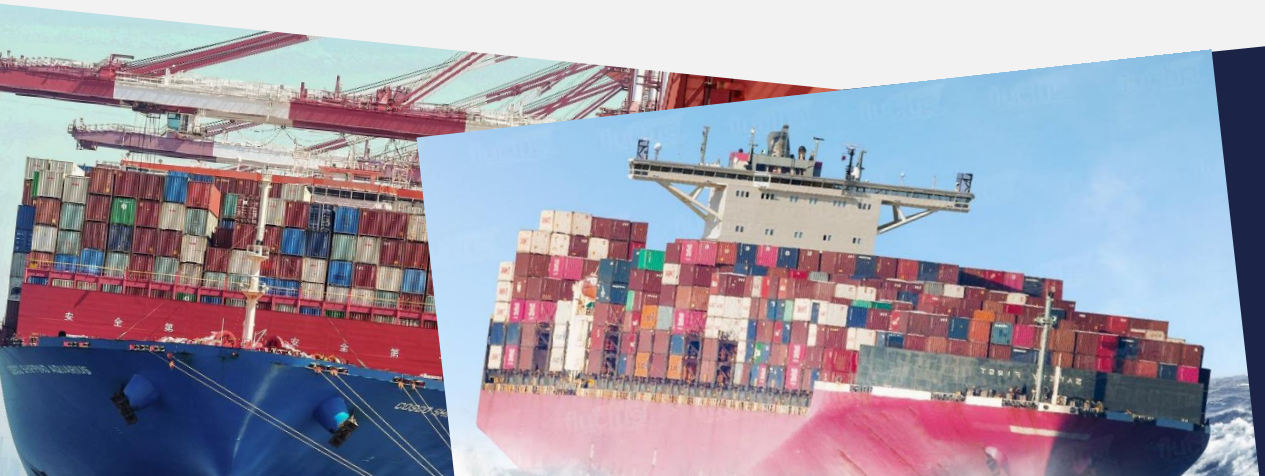





Scalable Stochastic Model for Sea Cargo Revenue Management with Empty Container Transportation

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Mikhail Nediak
Aliaksandr Nekrashevich
Mikhail Kovalyov
Genrikh Levin
Yuri Levin



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Research Motivation

1. Majority of cargo travels in containers by sea and rail
2. No model considering pricing and container management decisions together
3. Feature: empty container transportation
4. Inspiring results in Air Cargo modeling

Yuri Levin, Mikhail Nediak, and Huseyin Topaloglu. **"Cargo capacity management with allotments and spot market demand."** *Operations Research* 60.2 (2012): 351-365

5. Recent developments in subgradient optimization

Yuri Nesterov. (2015). **Universal gradient methods for convex optimization problems.** *Mathematical Programming*, *152*(1), 381-404





Related References

- Buckley, James J., and Lance C. Kendall. **“The business of shipping.” (2005)**
- **IVADO Labs**, project **ARGO for the Port of Montreal**. https://ivadolabs.com/en/projects/argo-for-the-port-of-montreal/
- Yuri Levin, Mikhail Nediak, and Huseyin Topaloglu. **"Cargo capacity management with allotments and spot market demand."** *Operations Research* 60.2 (2012): 351-365
- Yuri Nesterov. (2015). “**Universal gradient methods for convex optimization problems.”** *Mathematical Programming*, *152*(1), 381-404
- Trottier, Louis-Pierre and Cordeau, Jean-François. “**Solving the vessel routing and scheduling problem at a Canadian maritime transportation company,”** INFOR: Information Systems and Operational Research, vol. 57, no 2, 2019, p. 260-285.

Model Structure

- **Entities**

  container (TEU), port, vessel, itinerary, carrier, shipper



- **Phase Structure**

  Allotment Phase [allotments] + Spot Market Phase [bookings]



- **Event Structure**

  pricing, cut-off, arrival

Type	Pricing	Pricing	Pricing	Cutoff	Cutoff	Pricing	Arrival	Cutoff	Arrival	Cutoff	Arrival	Arrival
Real Time	83.00	85.00	89.00	97.23	99.88	101.00	103.09	103.34	106.76	115.49	118.93	126.03

Problem Type:

Markov Decision Process (allowing model-based approach), the objective is to optimize total expected profit



Typical Network

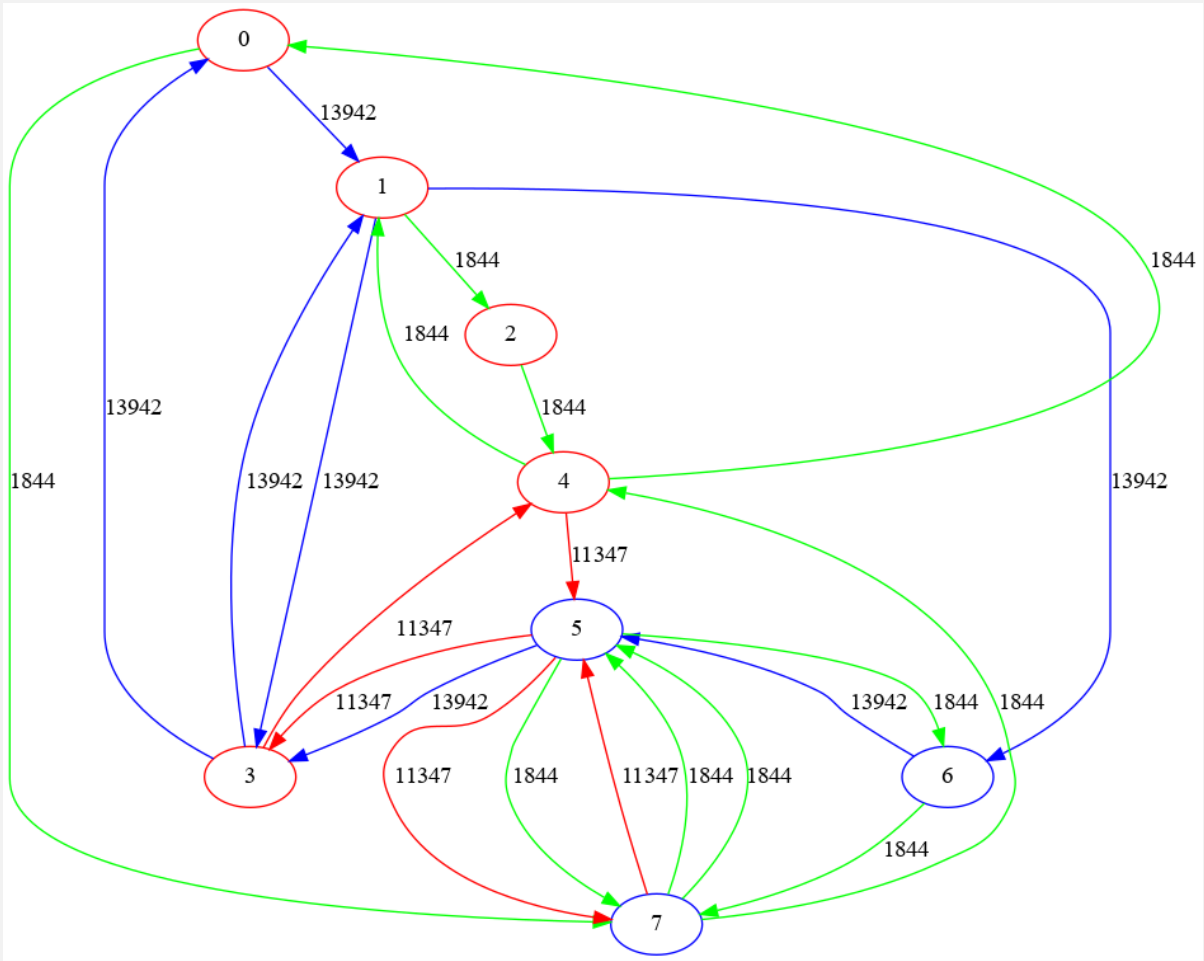








Artificial-P8-V3-Linear



Decision and State

Decision Structure

- Allotment Decision: $u^i, i \in I$
- Pricing: $p_{t,\sigma}$
- Loading and Declines at Cut-off: $Q_{r,\sigma}, Z_{r,\sigma}, Q^i_{r,\sigma}$
- Storing, hiring and off-hiring containers: $y_{H,\sigma}, y_{O,\sigma}, y_{s,\sigma}$



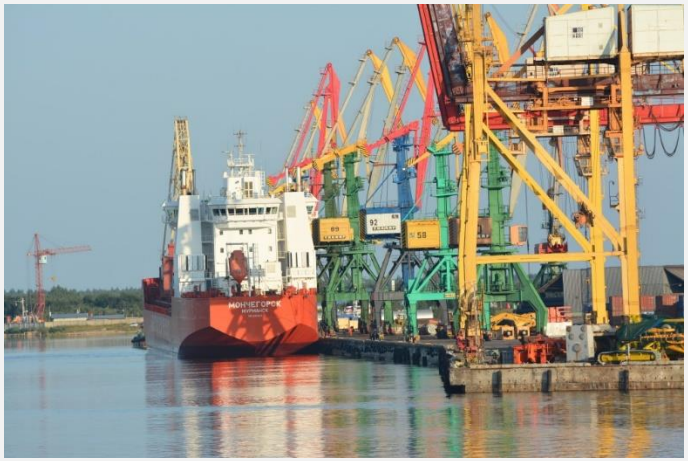


State Description

- Allotments
- Bookings
- Assigned Containers
- Empty containers on liner
- Containers In Ports

Major Challenges

- State Size (1000s of dimensions, 10000s TEU per dimension)
- Network Structure
- Alternation of Pricing, Arrival, and Cut-Off Events
- Extracting Profit from Empty Container Relocation

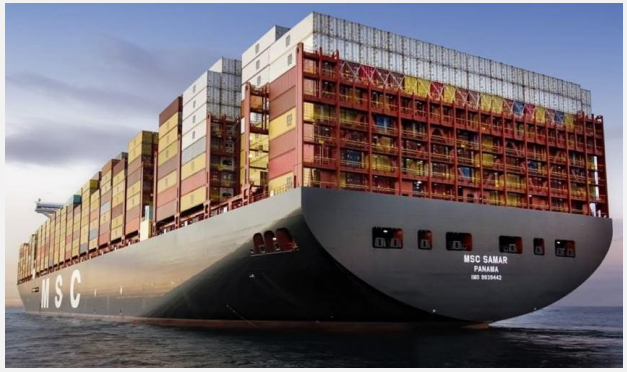




Objective Structure

$$(E^* \max\ *)^* [\text{nonlinear} + \text{linear}]$$

- **Nonlinearity Type:** $p_{t,\sigma} \cdot B_{t,\sigma}(p_{t,\sigma}), [\dots]^+, u^i(Q^i_{r,\sigma} + \dots)$





Constraint Structure

1. **Capacity Constraints** (TEU of the vessel)
2. Constraints on the **particular container type**
3. **Weight constraints**
4. **Hub Constraints** (TEU balance in ports)

LR-Framework – Constraint Relaxation

1. Base from Fluid Approach - **Freeze Hub Variables**









Decisions: TEU traveling on route F_r, TEU stored s_t, TEU hired/off-hired

2. Additional Flexibility - **Expand Decision Space**



Decisions: hired for route only, off-hired for route only

$$Q_r^{spot} + Q_r^{allotment} + Z_r^{empty} = F_r + y_r^H - y_r^O$$

Flexibility Balance

3. Lagrangian Relaxation - **Relax Weight/Capacity Constraints**



$$\lambda_a(Q^{spot} + \mathrm{Q}^{allotment} + \mathrm{Z}^{empty} - \mathrm{V_a})$$

$$\theta_a(Q^{spot} + \mathrm{Q}^{allotment} + \mathrm{Z}^{empty} - \mathrm{W_a})$$



LR-Framework – Decomposition and Optimization

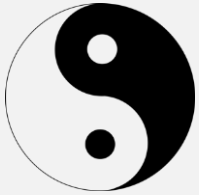
4. Decompose to single-itinerary problems



$$v_{t,r,\sigma}^{\lambda,\theta}(b_{r,\sigma}) \Longrightarrow$$

reasonable state,
computationally tractable

5. Approximate with Dual Linear Programming Problem Solution

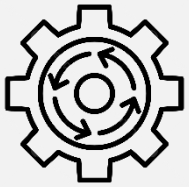


$$\mu_r(Q_r^{spot} + Q_r^{allotment} + Z_r^{empty} - F_r + y_r^H - y_r^O)$$

μ_r - dual variable to flexibility balance

Heuristic: μ_r is independent on the allotments and demand realization

6. Optimize using modern optimization methods



- Explicit convex piecewise-linear form for Binomial show realization
- Efficient with modern subgradient optimization methods (e.g. Nesterov UFGM)
- Spot Market Decision: prices provided at optimization, local linear maximization problem at cut-off stage

Yuri Nesterov. (2015). "**Universal gradient methods for convex optimization problems.**" *Mathematical Programming*, *152*(1), 381-404

Algorithms

Algorithm = Spot Strategy + Allotment Strategy



Spot Strategies:

- Greedy Baseline
- Fluid Approximation
- Lagrangian Relaxation

Allotment Strategies:

- Greedy Baseline
- Fluid Approximation





Underlined entries are LR-Framework based

Experiment Setup

Demand Types

- Linear
- Exponential



Set Types

- Maersk
- Artificial



Computed Metrics

- Overall Profit
- Spot Net Profit
- Allotment Net Profit
- Empty TEU
- Booked TEU
- Shipped TEU
- Computation Time

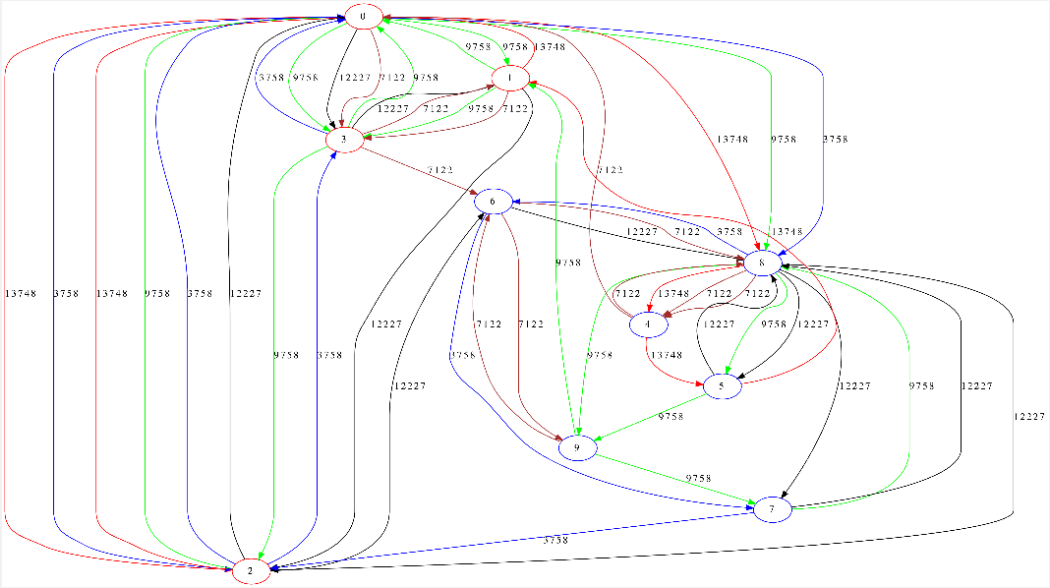
Experiment Summary

- Around 30 datasets
- Up to 25 market realizations
- 3 algorithms: (Greedy, Fluid, Lagrangian)
- Only TEU capacity constraints



Experiment: Artificial-P10-V5-Exponential

Parameter	Value
Number of Ports	10
Number of Vessels	5
Number of Arcs	112
Number of Allotments	198
Number of Itineraries	167
Number of Events	146
Time Spread	119
Time Before Shipping	30
Number of Pricing Decisions	334
Number of Travel Arcs	48
Number of Dual Variables	215
Number of Primal Variables	1491





Artificial-P10-V5-Exponential: Table With Results

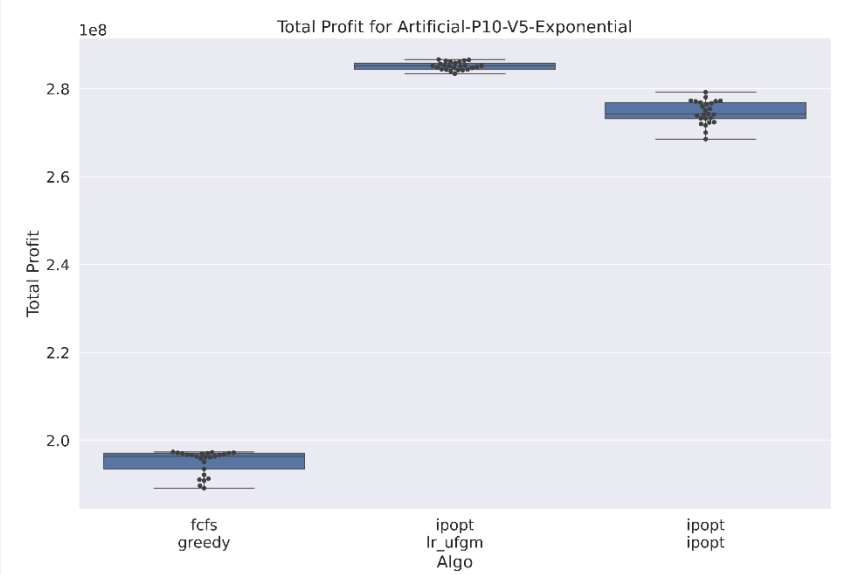
Algorithm	Empty TEU	Spot TEU	Allotment TEU	Spot Bookings	Allotment Bookings
Baseline	0.0	2320.44	118770.32	2496.92	128509.0
Lagrangian	22575.12	6083.44	112854.04	6603.4	122466.0
Fluid	22261.16	5865.0	112348.68	6505.6	122466.0
Algorithm	**Full Profit**	**Spot Profit**	**Allotment Profit**	**Container Cost**	**Empty Cost**
Baseline	195082262.86	5119802.54	332676095.25	-142713634.92	0.0
Lagrangian	285153565.12	6627792.24	344561740.02	-65617829.07	-418138.07
Fluid	274610649.08	4987919.80	335126819.37	-65092021.19	-412068.90



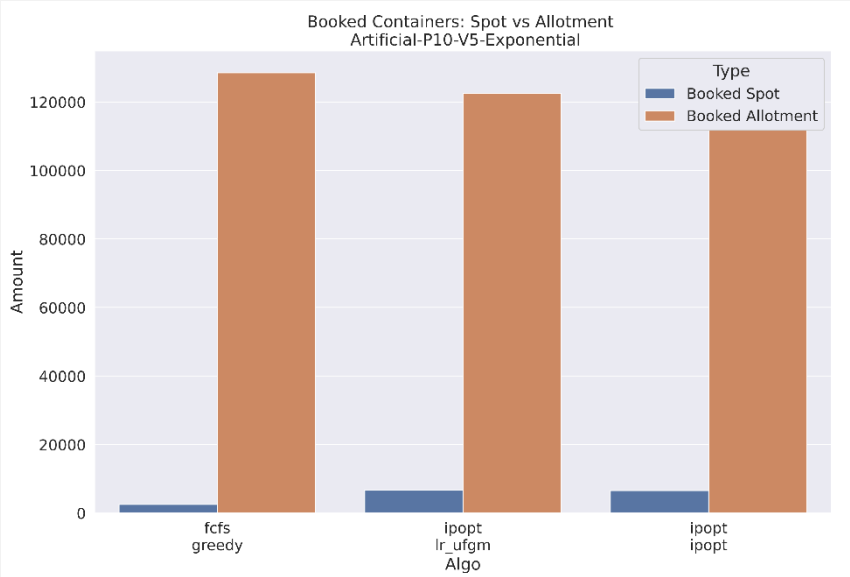


Artificial-P10-V5-Exponential: Profit and Shipped

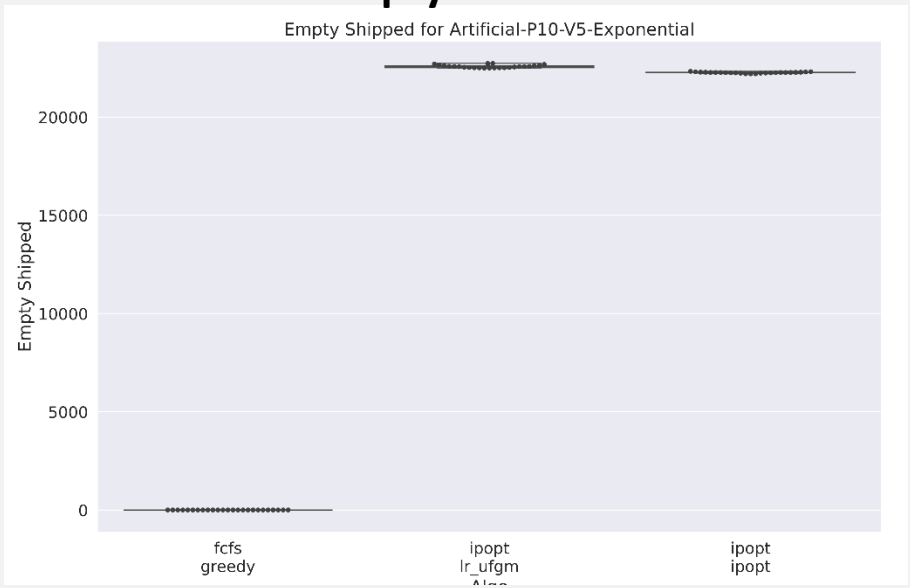
Total Profit



Booked: Spot vs Allotment

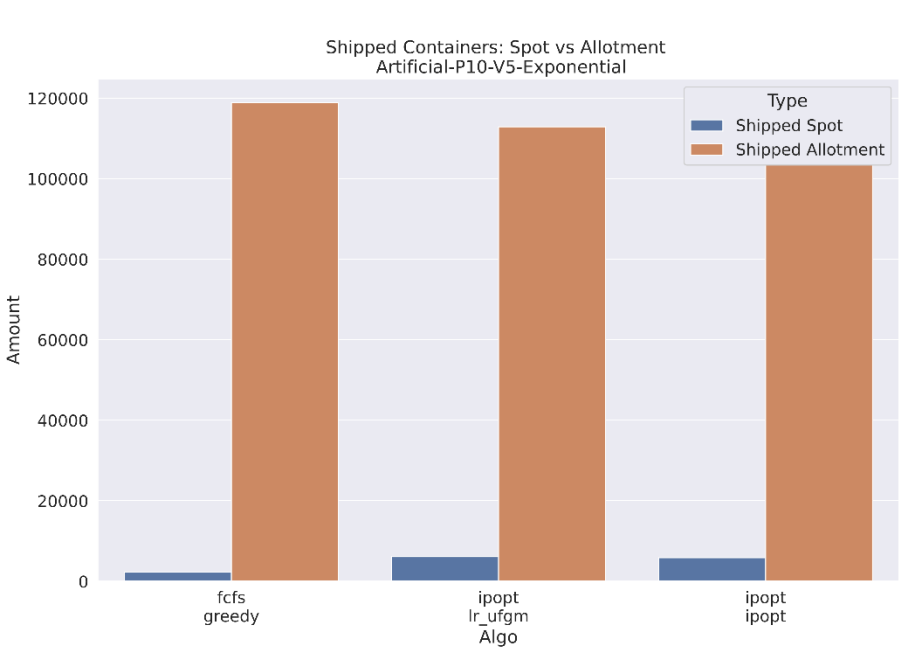


Empty Containers

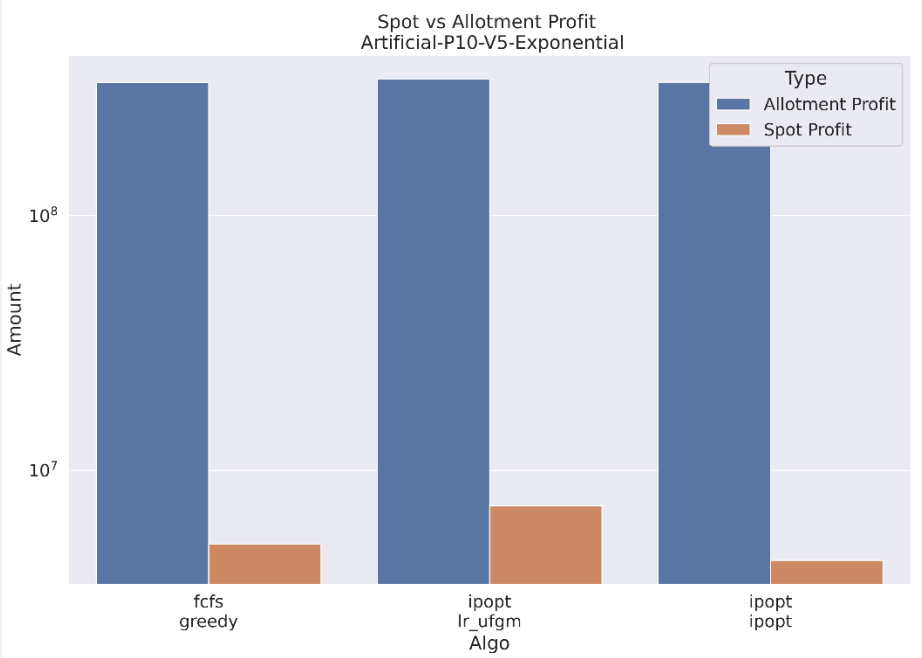


Artificial-P10-V5-Exponential: Profit and Shipped

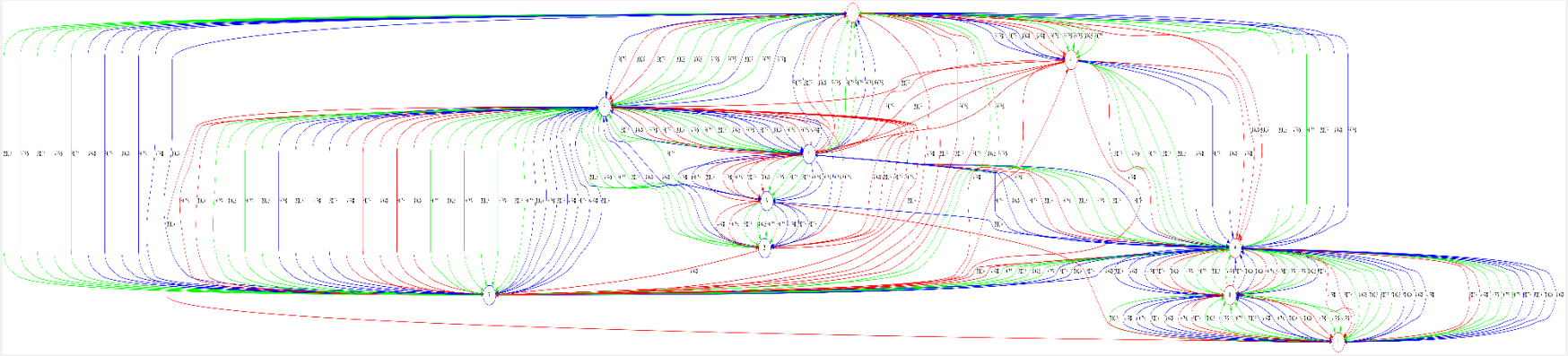
Number of Containers Shipped



Spot and Long Profit



Experiment: Maersk-P10-V3-Linear

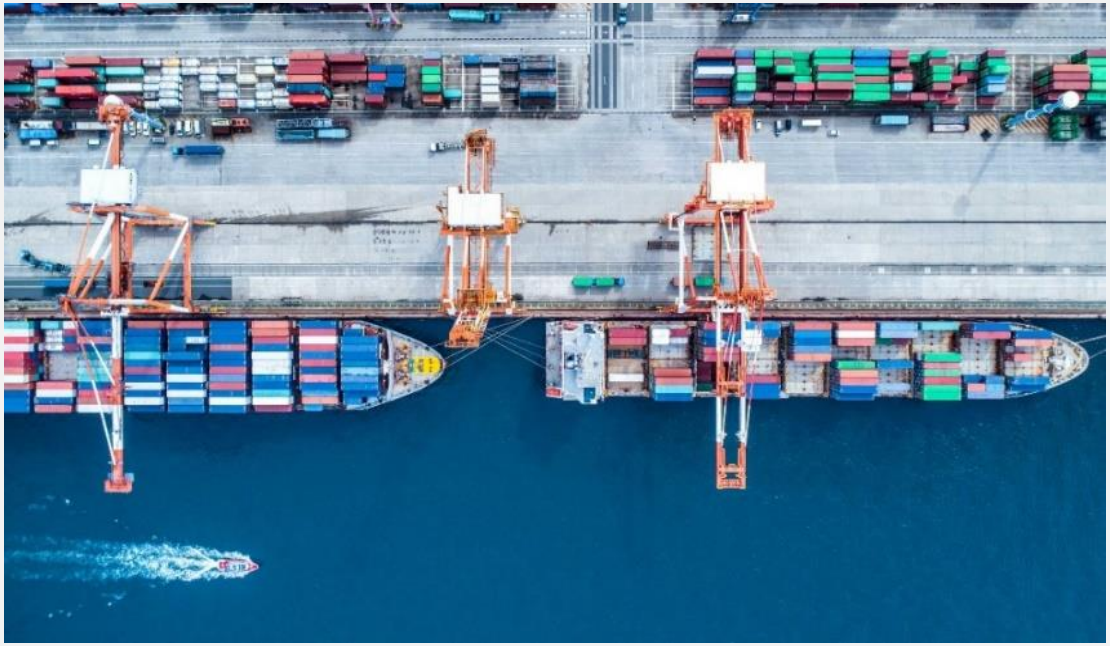


Network Visualization

Algorithm	Shipped Empty TEU	Shipped Spot TEU	Shipped Allotment TEU	Booked Spot TEU	Booked Allotment TEU
Lagrangian	32111.67	15718.58	140335.58	17477.25	151832.0
Fluid	32107.08	15045.5	139449.92	16661.5	151832.0
Algorithm	**Full Profit**	**Spot Profit**	**Allotment Profit**	**Container Profit**	**Empty Profit**
Lagrangian	63499108.50	748747.54	76872403.32	-13240974.48	-881067.88
Fluid	61731986.42	625634.08	73644559.37	-11663117.94	-875089.10

Summary

- **Novel stochastic scalable model** for making operational decisions: Dynamic Pricing, Lagrangian Relaxation, Interspersed Events, Container Allocation, Empty Containers.
- **Extended LR-Framework**: arbitrary event order, hub and capacity constraints together, effective subgradient optimization.



Thank you for your attention!

Sea Cargo Appendix. Model Formulae

- **Pricing:**

$$J_t^1(\mathbf{C}, \mathbf{b}_t^1, \mathbf{n}_t^1, \mathbf{y}_t^1, \boldsymbol{\varphi}_t^1) = \max_{\mathbf{p_t} \in \mathcal{P}_t} \mathop{\mathbb{E}}_{\mathbf{B}_t(\mathbf{p}_t)} [\mathbf{p}_t^T \cdot \mathbf{B}_t(\mathbf{p}_t) + J_t^2(\mathbf{C}, \mathbf{b}_t^1 + \mathbf{B}_t(\mathbf{p}_t), \mathbf{n}_t^1, \mathbf{y}_t^1, \boldsymbol{\varphi}_t^1)]$$

- **Cut-Off:**

$$J_t^1(\mathbf{C}, \mathbf{b}_t^1, \mathbf{n}_t^1, \mathbf{y}_t^1, \boldsymbol{\varphi}_t^1) = \mathop{\mathbb{E}}_{\mathbf{N}_a, \mathbf{N}_a^i} \Bigg[\max_{y_a^H, \mathbf{Q}_a^i, \mathbf{Z}_a, \mathbf{Q}_a} \Big\{ -\mathbf{m}_a^{B\intercal} \mathbf{b}_t^1 + \mathbf{N}_a^\intercal (-\mathbf{c}^D + \mathbf{m}_a^B)$$

$$+ \mathbf{Q}_a^\intercal \left(-\mathbf{c}^R + \mathbf{c}^D - f_a \sum_{\sigma \in \Omega} c_{g,\sigma}^S \mathbf{h}_\sigma \right)$$

$$+ \sum_{i \in I} u^i \left(\mathbf{N}_a^{i\intercal} (-\mathbf{p}_n^i - \mathbf{c}^D) + \mathbf{Q}_a^{i\intercal} (\mathbf{p}^i - \mathbf{c}^R + \mathbf{c}^D - f_a \sum_{\sigma \in \Omega} c_{g,\sigma}^S \mathbf{h}_\sigma) + \mathbf{A}_a^{i\intercal} \mathbf{p}_n^i \right)$$

$$- \mathbf{Z}_a^\intercal \mathbf{c}_e^R - \sum_{\sigma \in \Omega} c_{g,\sigma}^H y_{a,\sigma}^H + J_t^2 \left(\mathbf{C}, \mathbf{b}_t^1, \mathbf{n}_t^1 + \sum_i \mathbf{Q}_a^i + \mathbf{Z}_a + \mathbf{Q}_a, \right.$$

$$\left. \left. \mathbf{y}_t^1 + \sum_{\sigma \in \Omega} \mathbf{e}_{g,\sigma} \cdot \left(y_{a,\sigma}^H - (\sum_{i \in I} \mathbf{Q}_a^i + \mathbf{Z}_a + \mathbf{Q}_a)^\intercal \mathbf{h}_\sigma \right), \boldsymbol{\varphi}_t^1 + \mathbf{Z}_a \right) \right\} \Bigg]$$



Sea Cargo Appendix. Model Formulae

- **Arrival:**

$$J_t^1(\mathbf{C}, \mathbf{b}_t^1, \mathbf{n}_t^1, \mathbf{y}_t^1, \boldsymbol{\varphi}_t^1) = J_t^2(\mathbf{C}, \mathbf{b}_t^1, \mathbf{n}_t^1, \mathbf{y}_t^1 + \mathbf{M}_t \mathbf{n}_t^1, \boldsymbol{\varphi}_t^1)$$



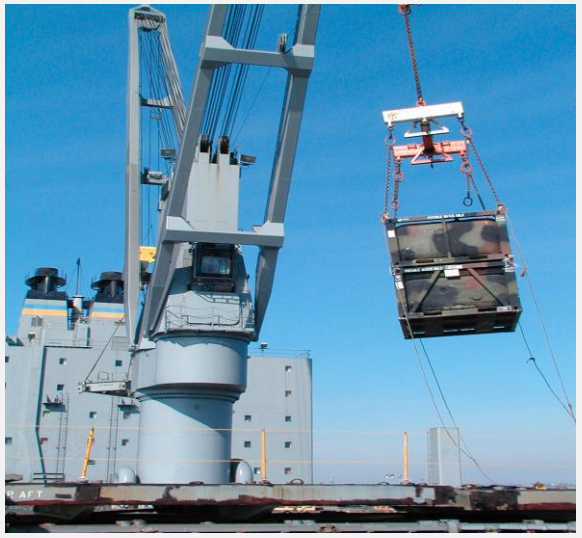
- **Constraints:**

$$\begin{cases} 0 \le \mathbf{Q}_a \le \mathbf{N}_a, \forall a \\ 0 \le \mathbf{Q}_a^i \le \mathbf{N}_a^i, \forall a, i \\ \mathbf{N}_{a,r} = 0, \ if \ start(r) \ne a, \forall a, r \\ \mathbf{Z}_a \ge 0, \forall a \\ \sum\limits_{\sigma \in \Omega} \left(\chi_\sigma \times \sum\limits_{r \in \mathcal{R}: b \in r} [Q_{r,\sigma} + Z_{r,\sigma} + \sum\limits_{i \in I} u^i Q_{r,\sigma}^i] \right) \le V_b, \forall b \\ \sum\limits_{r \in \mathcal{R}: b \in r} [Q_{r,\sigma} + Z_{r,\sigma} + \sum\limits_{i \in I} u^i Q_{r,\sigma}^i] \le V_{b,\sigma}, \forall b, \forall \sigma \in \Omega \\ \sum\limits_{\sigma \in \Omega} \left(\sum\limits_{r \in \mathcal{R}: b \in r} [\omega_\sigma \times Q_{r,\sigma} + \omega_\sigma^e \times Z_{r,\sigma} + \omega_\sigma \times \sum\limits_{i \in I} u^i Q_{r,\sigma}^i] \right) \le W_b, \forall b \\ y_{t,g,\sigma}^1 + y_{a,\sigma}^H - (\sum_i u^i \mathbf{Q}_a^i + \mathbf{Z}_a + \mathbf{Q}_a)^\intercal \mathbf{h}_\sigma \ge 0, \forall \sigma \in \Omega, \forall \text{ cut-off } a = (g,t) \to (g',t') \end{cases}$$

Sea Cargo Appendix. Model Formulae

- **Optimization Problem:**

$$J_0(\mathbf{C}) = \max_{u_i} J_1^1(\mathbf{C}, \mathbf{0}, \mathbf{0}, \mathbf{0}, \mathbf{0})$$



- **Group Constraints:**

$$\begin{cases} \sum_{i \in I_j} u^i \leq 1, \forall j \\ u^i \in \{0, 1\} \end{cases}$$

- **Container Allocation:**

$$J_t^2(\mathbf{C}, \mathbf{b}_t^2, \mathbf{n}_t^2, \mathbf{y}_t^2, \boldsymbol{\varphi}_t^2) = \max_{0 \leq \mathbf{s}_t \leq \mathbf{y}_t^2} -d_t(\mathbf{y}_t^2 - \mathbf{s}_t)^\intercal \mathbf{c}^S - \mathbf{s}_t^T \mathbf{c}^O + J_{t+1}^1(\mathbf{C}, \mathbf{b}_t^2, \mathbf{n}_t^2, \mathbf{y}_t^2 - \mathbf{s}_t, \boldsymbol{\varphi}_t^2)$$



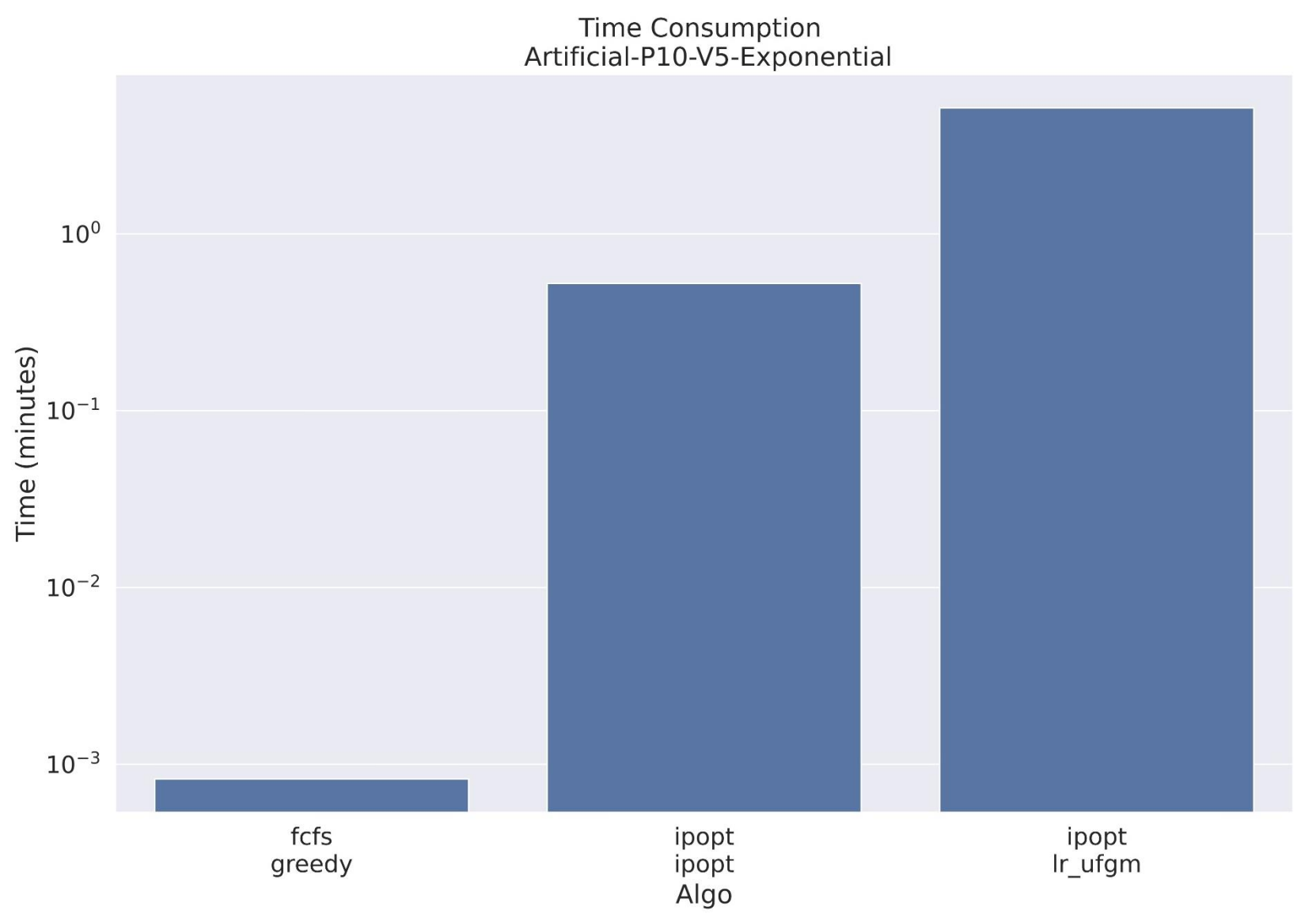


Sea Cargo Appendix. Decomposition Structure

Theorem. *Two-Level Relaxed Dynamic Program is separable in variables related to routes and service types (i.e. in* $(Q_{r,\sigma}, Z_{r,\sigma}, N_{r,\sigma}, F_{r,\sigma}, Q^i_{r,\sigma}, N^i_{r,\sigma}, A^i_{r,\sigma}, y^{HS}_{r,\sigma}, y^{OS}_{r,\sigma}, b_{t,r,\sigma}, n_{t,r,\sigma}, \varphi_{t,r,\sigma})$*), given that* λ_b, θ_b, $\lambda_{b,\sigma}$ *are fixed. Moreover, the following decomposition holds for* $t \geq 1$*:*

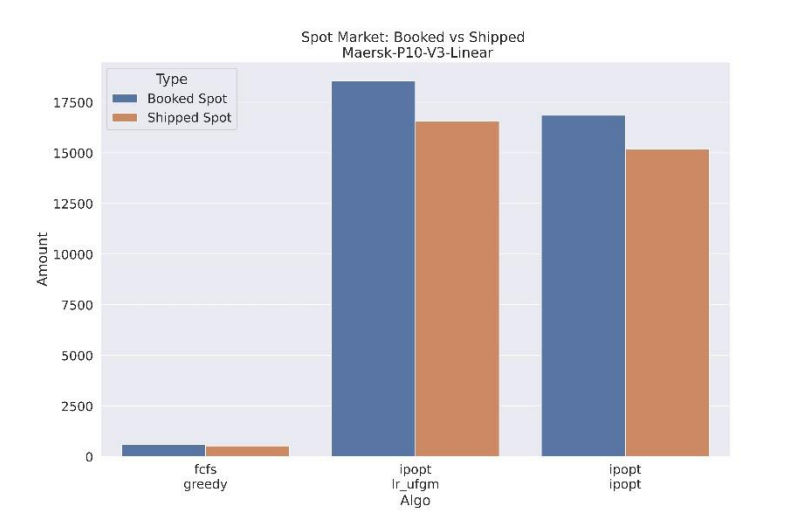
$$
\begin{aligned}
&KR_t^{1,\boldsymbol{\lambda},\boldsymbol{\theta},F,s,y^H}(\mathbf{C}, \mathbf{b}_t^1, \mathbf{n}_t^1, \mathbf{y}_t^1, \mathbf{l}_t^1, \boldsymbol{\varphi}_t^1) = \\
&\quad - \sum_{r:T_{start(r)}<t} \sum_{\sigma\in\Omega} \left((l^1_{t,r,\sigma} + n^1_{t,r,\sigma}) \sum_{b\in r} [\lambda_b \times \chi_\sigma + \theta_b \times \omega_\sigma + \lambda_{b,\sigma}] \right) \\
&\quad - \sum_{r:T_{strat(r)<t}} \sum_{\sigma\in\Omega} \left(w_\sigma \times (l^1_{t,r,\sigma} + n^1_{t,r,\sigma} - \varphi^1_{t,r,\sigma}) + w^e_\sigma \times \varphi^1_{t,r,\sigma} \right) \times \left(\sum_{b\in r} \theta_b \right) \\
&\quad + \sum_{r\in\mathcal{R}} \sum_{\sigma\in\Omega} v^{\boldsymbol{\lambda},\boldsymbol{\theta}}_{t,r,\sigma}(b_{r,\sigma}) + \sum_{b:\ travel} (\lambda_b V_b + \theta_b W_b + \sum_{\sigma\in\Omega} \lambda_{b,\sigma} V_{b,\sigma}) \\
&- \sum_{a=(g,t^*)\to(g',t'),T_a\geq t} \sum_{\sigma\in\Omega} c^H_{g,\sigma} y^H_{a,\sigma} - \sum_{T\geq t'\geq t} \mathbf{c}^{O\intercal}\mathbf{s}_{\mathbf{t}'} - \sum_{T\geq t'\geq t} d_{t'} \mathbf{y}^{1\intercal}_{t'+1} \mathbf{c}^S - \mathbf{c}^{O\intercal}\mathbf{y}^1_{T+1}
\end{aligned}
$$

Sea Cargo Appendix.
Artificial-P10-V5-Exponential: Execution Time

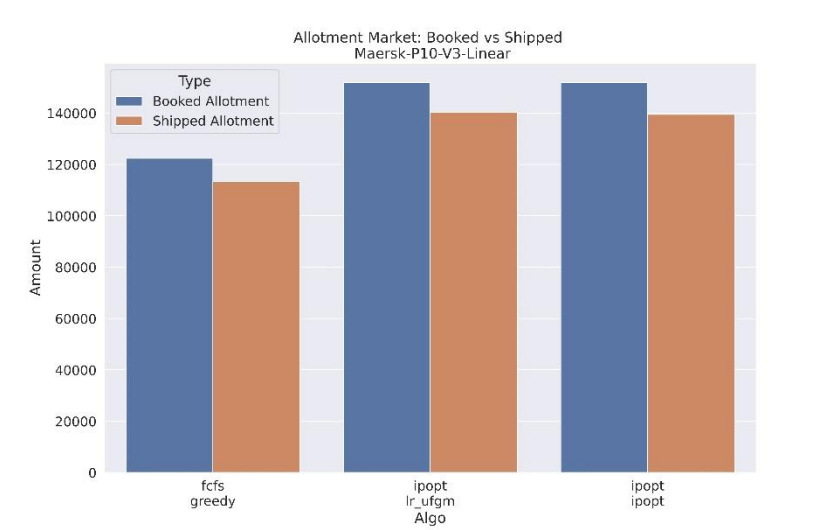


Sea Cargo Appendix.
Maersk-P10-V3-Linear: Containers and Profit

Spot Market: Booked vs Shipped



Allotment Market: Booked vs Shipped



Spot vs Allotment Profit

