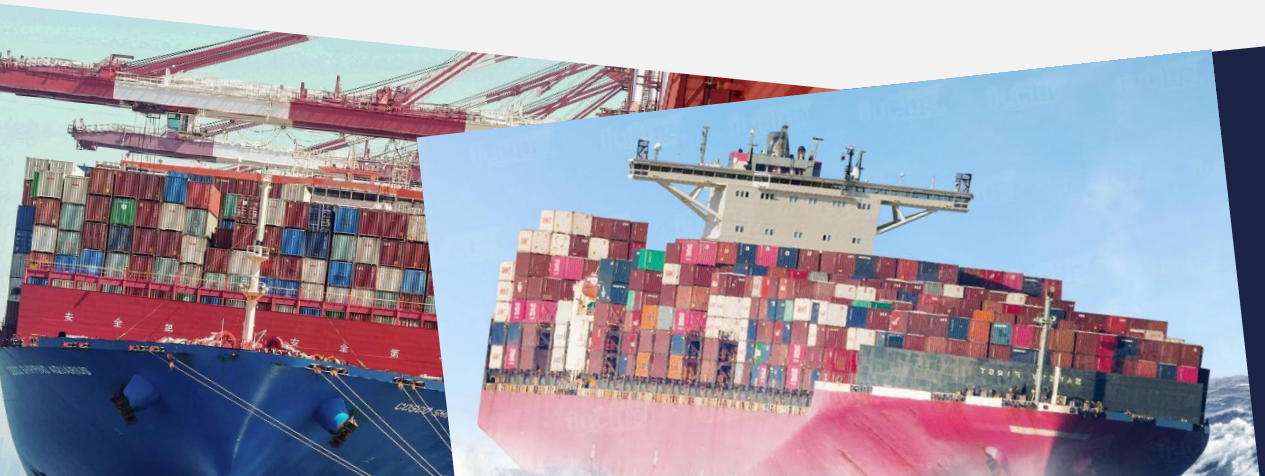





Scalable Stochastic Model for Sea Cargo Revenue Management with Empty Container Transportation

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Mikhail Nediak
Aliaksandr Nekrashevich
Mikhail Kovalyov
Genrikh Levin
Yuri Levin



Smith School of Business | Queen's University

Research Motivation

1. Majority of cargo travels in containers by sea and rail
2. No model considering pricing and container management decisions together
3. Feature: empty container transportation
4. Inspiring results in Air Cargo modeling

   Yuri Levin, Mikhail Nediak, and Huseyin Topaloglu. **"Cargo capacity management with allotments and spot market demand."** *Operations Research* 60.2 (2012): 351-365

5. Recent developments in subgradient optimization

   Yuri Nesterov. (2015). **Universal gradient methods for convex optimization problems.** *Mathematical Programming*, *152*(1), 381-404





Related References

- Buckley, James J., and Lance C. Kendall. **“The business of shipping.” (2005)**
- **IVADO Labs**, project **ARGO for the Port of Montreal**. https://ivadolabs.com/en/projects/argo-for-the-port-of-montreal/
- Yuri Levin, Mikhail Nediak, and Huseyin Topaloglu. **"Cargo capacity management with allotments and spot market demand."** *Operations Research* 60.2 (2012): 351-365
- Yuri Nesterov. (2015). “**Universal gradient methods for convex optimization problems.”** *Mathematical Programming*, *152*(1), 381-404
- Trottier, Louis-Pierre and Cordeau, Jean-François. “**Solving the vessel routing and scheduling problem at a Canadian maritime transportation company,”** INFOR: Information Systems and Operational Research, vol. 57, no 2, 2019, p. 260-285.

Model Structure

- **Entities**

  container (TEU), port, vessel, itinerary, carrier, shipper



- **Phase Structure**

  Allotment Phase [allotments] + Spot Market Phase [bookings]



- **Event Structure**

  pricing, cut-off, arrival

Type	Pricing	Pricing	Pricing	Cutoff	Cutoff	Pricing	Arrival	Cutoff	Arrival	Cutoff	Arrival	Arrival
Real Time	83.00	85.00	89.00	97.23	99.88	101.00	103.09	103.34	106.76	115.49	118.93	126.03

Problem Type:

Markov Decision Process (allowing model-based approach), the objective is to optimize total expected profit



Typical Network

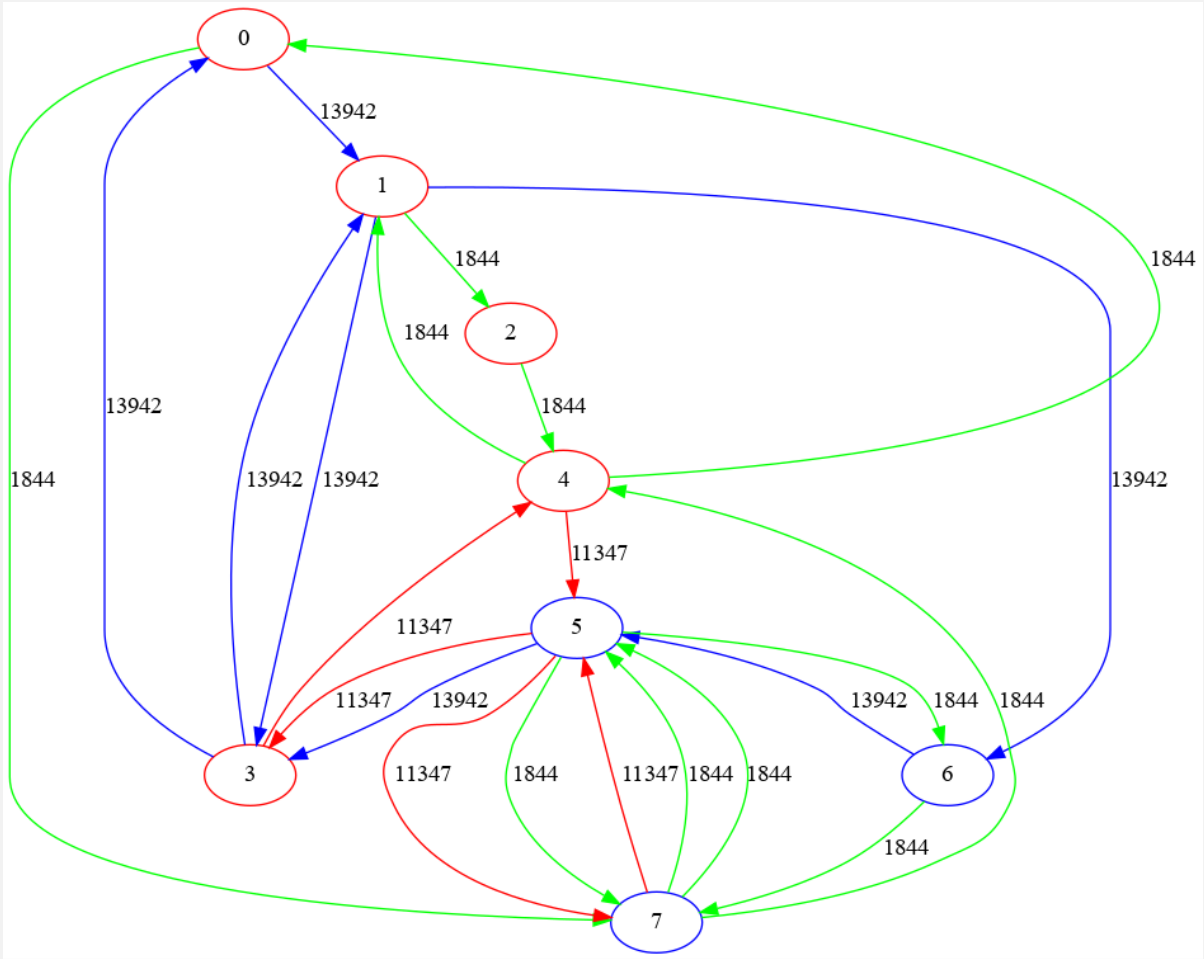








Artificial-P8-V3-Linear



Decision and State

Decision Structure

- Allotment Decision: $u^i, i \in I$
- Pricing: $p_{t,\sigma}$
- Loading and Declines at Cut-off: $Q_{r,\sigma}, Z_{r,\sigma}, Q^i_{r,\sigma}$
- Storing, hiring and off-hiring containers: $y_{H,\sigma}, y_{O,\sigma}, y_{s,\sigma}$



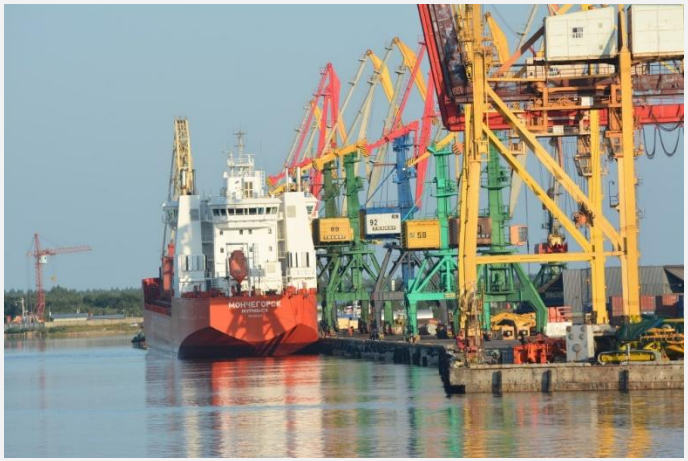


State Description

- Allotments
- Bookings
- Assigned Containers
- Empty containers on liner
- Containers In Ports

Major Challenges

- State Size (1000s of dimensions, 10000s TEU per dimension)
- Network Structure
- Alternation of Pricing, Arrival, and Cut-Off Events
- Extracting Profit from Empty Container Relocation

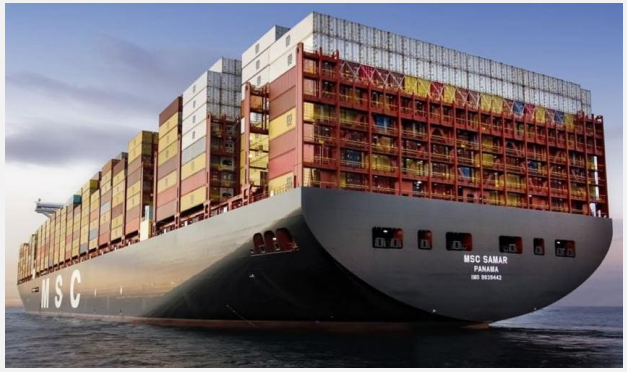




Objective Structure

$$(E^* \max\ *)^* [\text{nonlinear} + \text{linear}]$$

- **Nonlinearity Type**: $p_{t,\sigma} \cdot B_{t,\sigma}(p_{t,\sigma}), [\dots]^+, u^i(Q^i_{r,\sigma} + \dots)$





Constraint Structure

1. **Capacity Constraints** (TEU of the vessel)
2. Constraints on the **particular container type**
3. **Weight constraints**
4. **Hub Constraints** (TEU balance in ports)

LR-Framework

1. Expand Decision Space
2. Freeze Hub Variables
3. Relax Capacity Constraints
4. Decompose to single-itinerary problems
5. Approximate with Dual Linear Programming Problem Solution
6. Optimize using modern optimization methods





Algorithms

Algorithm = Spot Strategy + Allotment Strategy



Spot Strategies:

- Greedy Baseline
- Fluid Approximation
- Lagrangian Relaxation

Allotment Strategies:

- Greedy Baseline
- Fluid Approximation





Underlined entries are LR-Framework based

Experiment Setup

Demand Types

- Linear
- Exponential



Set Types

- Maersk
- Artificial



Computed Metrics

- Overall Profit
- Spot Net Profit
- Allotment Net Profit
- Empty TEU
- Booked TEU
- Shipped TEU
- Computation Time

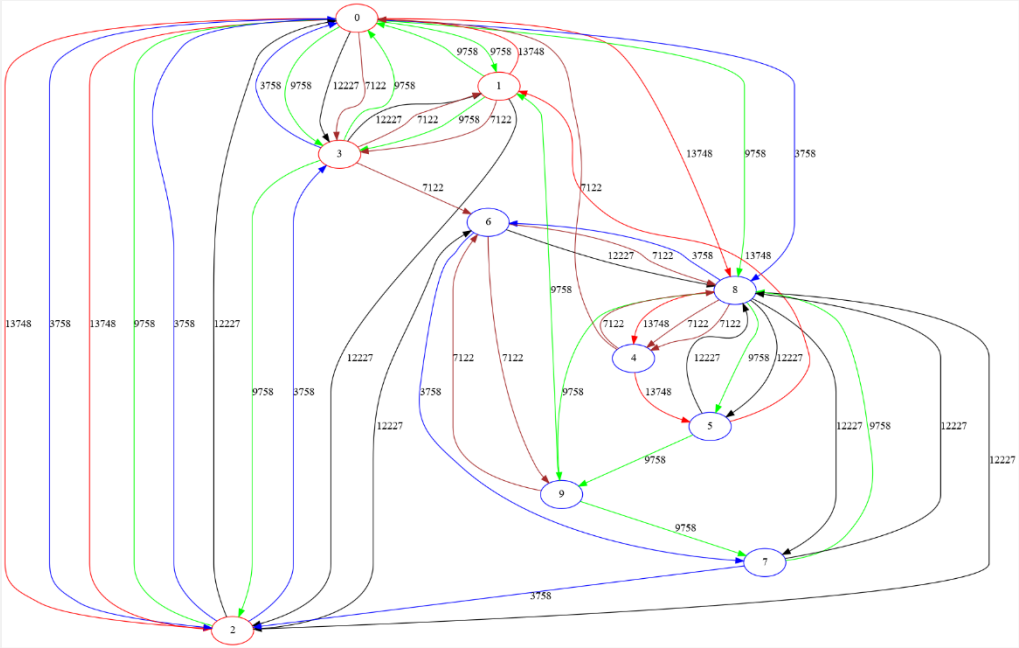
Experiment Summary

- Around 30 datasets
- Up to 25 market realizations
- 3 algorithms: (Greedy, Fluid, Lagrangian)
- Only TEU capacity constraints



Experiment: Artificial-P10-V5-Exponential

Description	Value
# Ports	10
# Ocean Liners	5
# Arcs	112
# Allotments	200
# Itineraries	167
# Pricing Events	54
# Cutoff Events	46
# Arrival Events	46
# Pricing Decisions	334
# Travel Arcs	48
# Dual Variables	215
# Primal Variables	1510





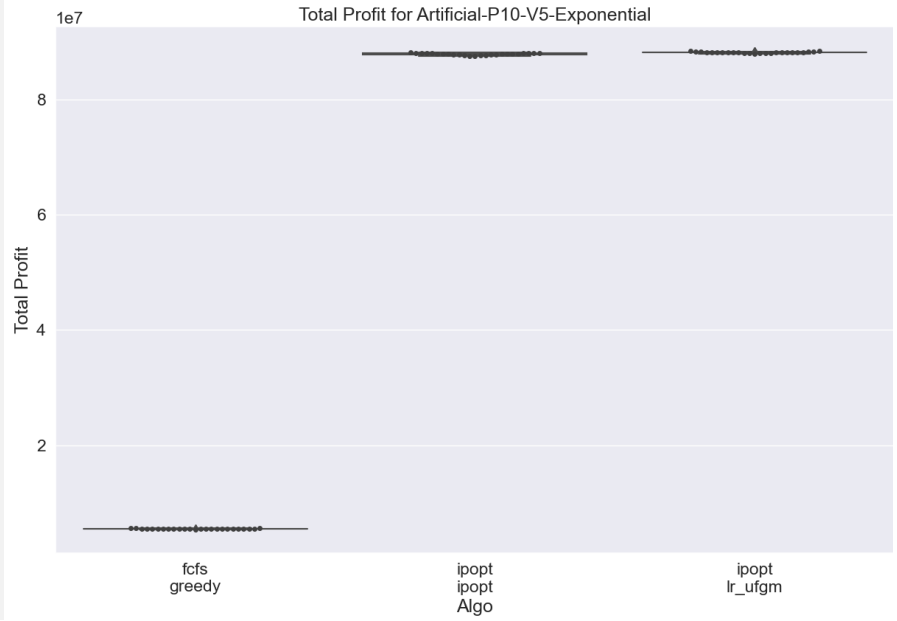
Artificial-P10-V5-Exponential: Table With Results

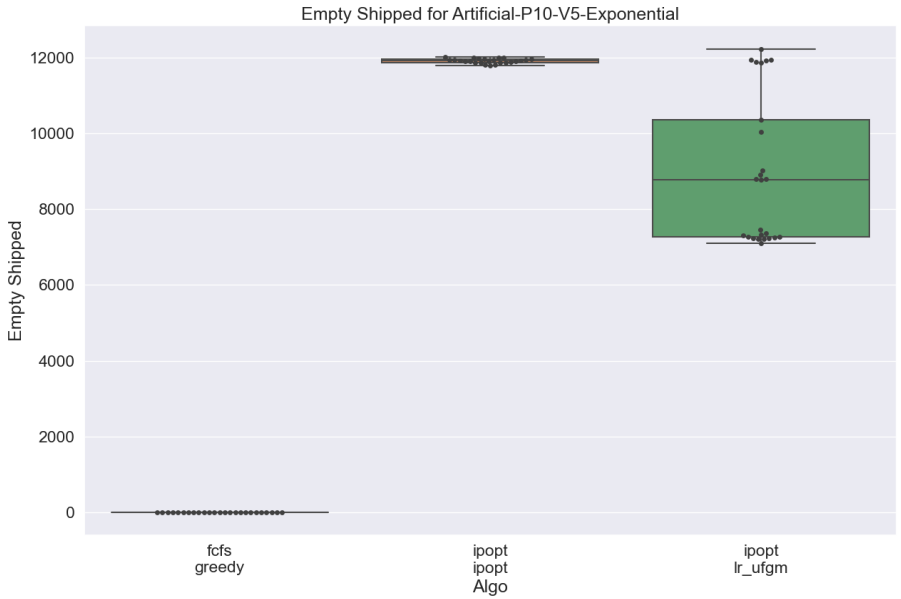
Algorithm	Empty #	Spot #	Allotment #	Bookings Spot #	Bookings Allotment #
Fluid	11925	15930	134301	19687	146699
Lagrangian	8955	17275	135104	20739	146699
Algorithm	**Profit $**	**Spot Profit $**	**Long Profit $**	**Containers $**	**Empty $**
Fluid	87849330.37	10833953.09	81787857.60	-4561754.18	-210726.13
Lagrangian	88137031.34	10925281.96	82246266.16	-4898840.49	-135676.27

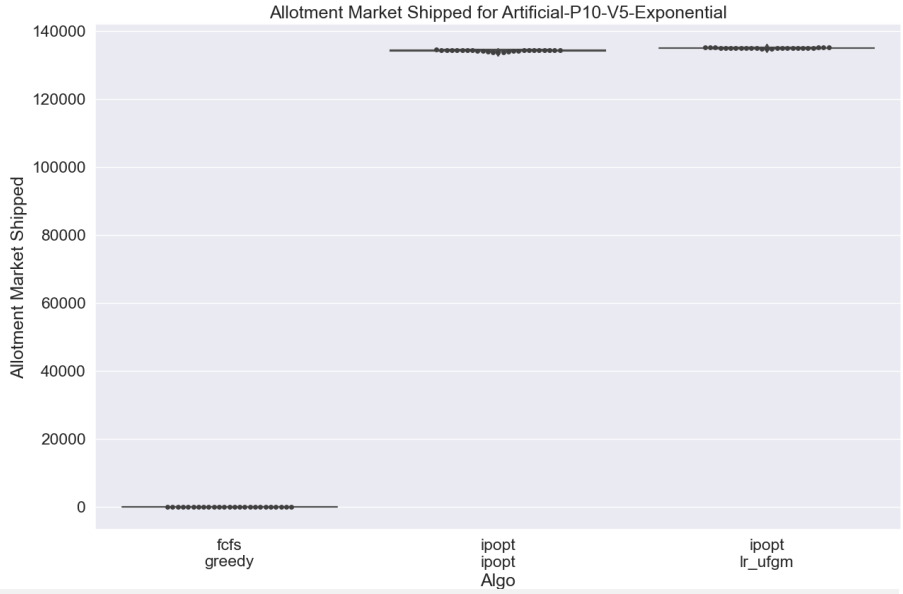


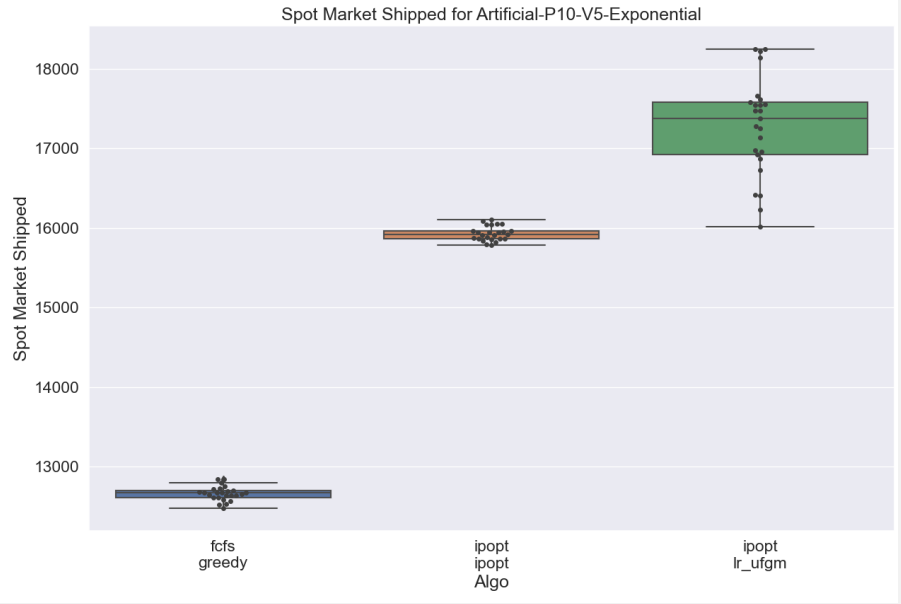


Artificial-P10-V5-Exponential: Profit and Shipped





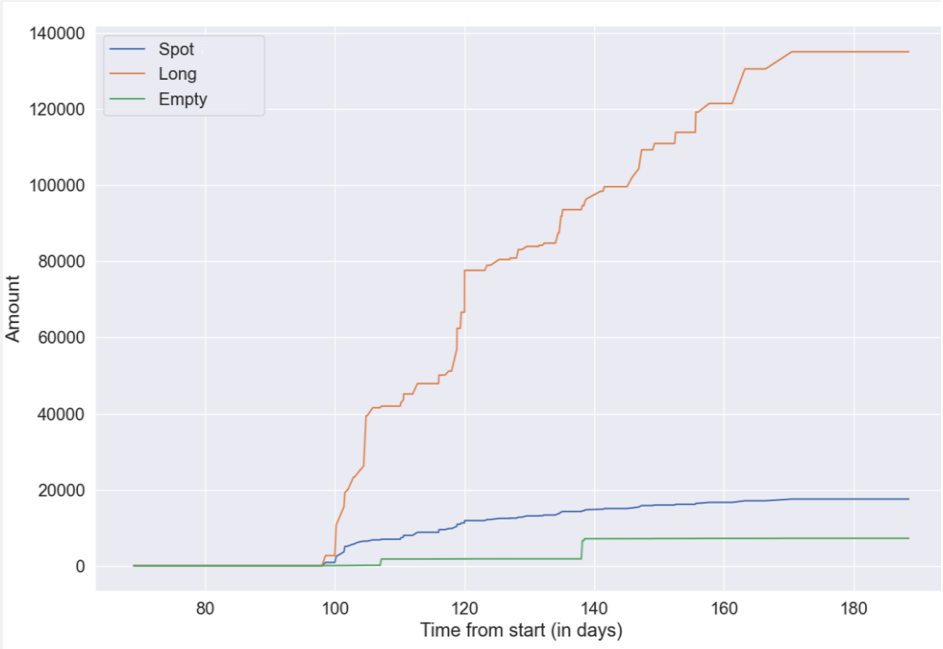




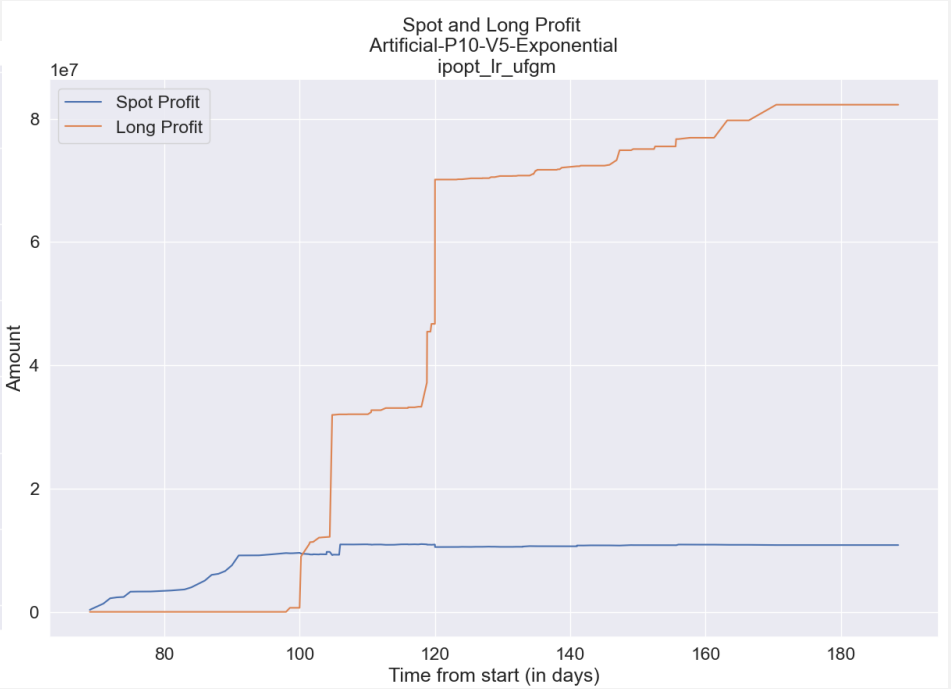
Artificial-P10-V5-Exponential: Profit and Shipped

Algorithm: IPOPT-LR-UFGM

Artificial-P10-V5-Exponential
Number of Containers Shipped

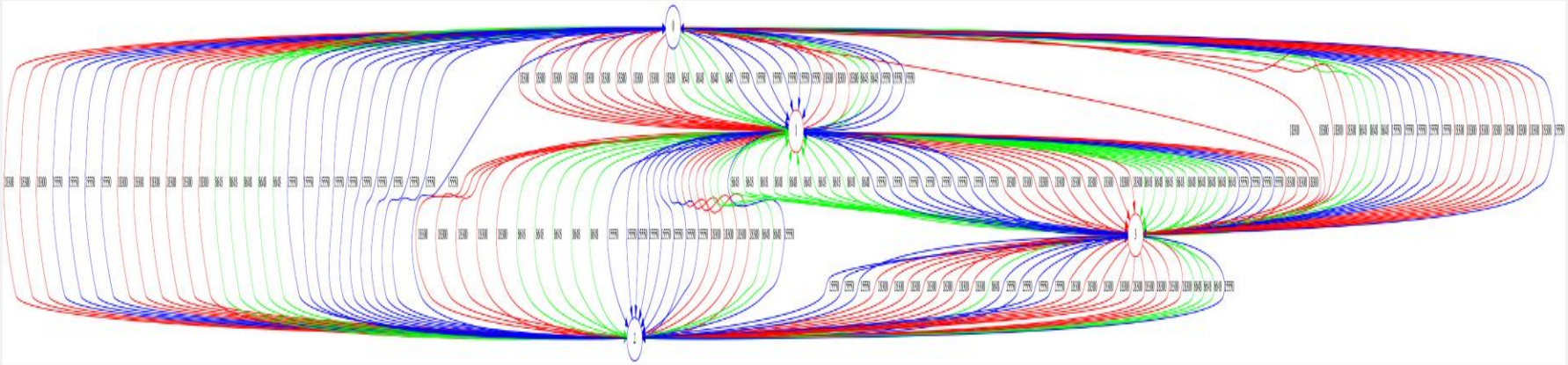


Artificial-P10-V5-Exponential
Spot and Long Profit



Experiment: Maersk-P4-V4-Linear

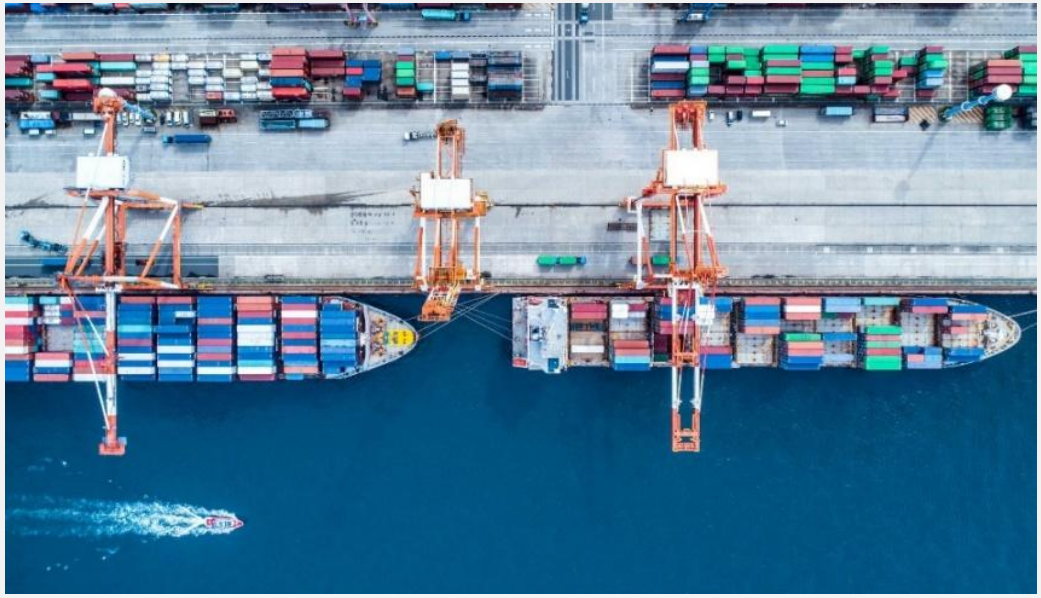
Network Visualization



Algorithm	# Empty TEU	# Spot TEU	# Allotment TEU	# Spot Bookings	# Allotment Bookings
Fluid	179315	166992	1083562	244523	1281815
Lagrangian	168531	171202	1082078	248560	1281815
Algorithm	**Profit $**	**$ Spot**	**$ Allotment**	**$ Container**	**$ Empty TEU**
Fluid	5191771871.5	656999873.4	4557024400.3	-18308105.9	-3944296.3
Lagrangian	5185027937.6	657421215.9	4551949828.9	-20459984.7	-3883122.4

Summary

- **Novel stochastic scalable model** for making operational decisions. Dynamic Pricing, Lagrangian Relaxation, Empty Containers.
- **Extended LR-Framework**: arbitrary event order, hub and capacity constraints together, effective subgradient optimization.



Thank you for your attention!

Sea Cargo Appendix. Model Formulae

- **Pricing:**

$$J_t^1(\mathbf{C}, \mathbf{b}_t^1, \mathbf{n}_t^1, \mathbf{y}_t^1, \boldsymbol{\varphi}_t^1) = \max_{\mathbf{p_t} \in \mathcal{P}_t} \mathop{\mathbb{E}}_{\mathbf{B}_t(\mathbf{p}_t)} [\mathbf{p}_t^T \cdot \mathbf{B}_t(\mathbf{p}_t) + J_t^2(\mathbf{C}, \mathbf{b}_t^1 + \mathbf{B}_t(\mathbf{p}_t), \mathbf{n}_t^1, \mathbf{y}_t^1, \boldsymbol{\varphi}_t^1)]$$

- **Cut-Off:**

$$J_t^1(\mathbf{C}, \mathbf{b}_t^1, \mathbf{n}_t^1, \mathbf{y}_t^1, \boldsymbol{\varphi}_t^1) = \mathop{\mathbb{E}}_{\mathbf{N}_a, \mathbf{N}_a^i} \Bigg[\max_{y_a^H, \mathbf{Q}_a^i, \mathbf{Z}_a, \mathbf{Q}_a} \Big\{ -\mathbf{m}_a^{B\intercal} \mathbf{b}_t^1 + \mathbf{N}_a^\intercal (-\mathbf{c}^D + \mathbf{m}_a^B)$$

$$+ \mathbf{Q}_a^\intercal \left(-\mathbf{c}^R + \mathbf{c}^D - f_a \sum_{\sigma \in \Omega} c_{g,\sigma}^S \mathbf{h}_\sigma \right)$$

$$+ \sum_{i \in I} u^i \left(\mathbf{N}_a^{i\intercal} (-\mathbf{p}_n^i - \mathbf{c}^D) + \mathbf{Q}_a^{i\intercal} (\mathbf{p}^i - \mathbf{c}^R + \mathbf{c}^D - f_a \sum_{\sigma \in \Omega} c_{g,\sigma}^S \mathbf{h}_\sigma) + \mathbf{A}_a^{i\intercal} \mathbf{p}_n^i \right)$$

$$- \mathbf{Z}_a^\intercal \mathbf{c}_e^R - \sum_{\sigma \in \Omega} c_{g,\sigma}^H y_{a,\sigma}^H + J_t^2 \left(\mathbf{C}, \mathbf{b}_t^1, \mathbf{n}_t^1 + \sum_i \mathbf{Q}_a^i + \mathbf{Z}_a + \mathbf{Q}_a, \right.$$

$$\left. \left. \mathbf{y}_t^1 + \sum_{\sigma \in \Omega} \mathbf{e}_{g,\sigma} \cdot \left(y_{a,\sigma}^H - (\sum_{i \in I} \mathbf{Q}_a^i + \mathbf{Z}_a + \mathbf{Q}_a)^\intercal \mathbf{h}_\sigma \right), \boldsymbol{\varphi}_t^1 + \mathbf{Z}_a \right) \right\} \Bigg]$$



Sea Cargo Appendix. Model Formulae

- **Arrival:**

$$J_t^1(\mathbf{C}, \mathbf{b}_t^1, \mathbf{n}_t^1, \mathbf{y}_t^1, \boldsymbol{\varphi}_t^1) = J_t^2(\mathbf{C}, \mathbf{b}_t^1, \mathbf{n}_t^1, \mathbf{y}_t^1 + \mathbf{M}_t \mathbf{n}_t^1, \boldsymbol{\varphi}_t^1)$$



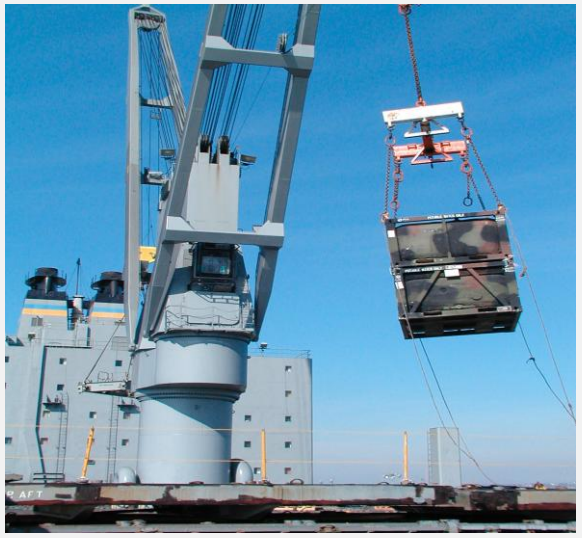
- **Constraints:**

$$\begin{cases} 0 \le \mathbf{Q}_a \le \mathbf{N}_a, \forall a \\ 0 \le \mathbf{Q}_a^i \le \mathbf{N}_a^i, \forall a, i \\ \mathbf{N}_{a,r} = 0, \ if \ start(r) \ne a, \forall a, r \\ \mathbf{Z}_a \ge 0, \forall a \\ \sum\limits_{\sigma \in \Omega} \left(\chi_\sigma \times \sum\limits_{r \in \mathcal{R}: b \in r} [Q_{r,\sigma} + Z_{r,\sigma} + \sum\limits_{i \in I} u^i Q_{r,\sigma}^i] \right) \le V_b, \forall b \\ \sum\limits_{r \in \mathcal{R}: b \in r} [Q_{r,\sigma} + Z_{r,\sigma} + \sum\limits_{i \in I} u^i Q_{r,\sigma}^i] \le V_{b,\sigma}, \forall b, \forall \sigma \in \Omega \\ \sum\limits_{\sigma \in \Omega} \left(\sum\limits_{r \in \mathcal{R}: b \in r} [\omega_\sigma \times Q_{r,\sigma} + \omega_\sigma^e \times Z_{r,\sigma} + \omega_\sigma \times \sum\limits_{i \in I} u^i Q_{r,\sigma}^i] \right) \le W_b, \forall b \\ y_{t,g,\sigma}^1 + y_{a,\sigma}^H - (\sum_i u^i \mathbf{Q}_a^i + \mathbf{Z}_a + \mathbf{Q}_a)^\intercal \mathbf{h}_\sigma \ge 0, \forall \sigma \in \Omega, \forall \text{ cut-off } a = (g,t) \to (g',t') \end{cases}$$

Sea Cargo Appendix. Model Formulae

- **Optimization Problem:**

$$J_0(\mathbf{C}) = \max_{u_i} J_1^1(\mathbf{C}, \mathbf{0}, \mathbf{0}, \mathbf{0}, \mathbf{0})$$



- **Group Constraints:**

$$\begin{cases} \sum_{i \in I_j} u^i \leq 1, \forall j \\ u^i \in \{0, 1\} \end{cases}$$

- **Container Allocation:**

$$J_t^2(\mathbf{C}, \mathbf{b}_t^2, \mathbf{n}_t^2, \mathbf{y}_t^2, \boldsymbol{\varphi}_t^2) = \max_{0 \leq \mathbf{s}_t \leq \mathbf{y}_t^2} -d_t(\mathbf{y}_t^2 - \mathbf{s}_t)^\intercal \mathbf{c}^S - \mathbf{s}_t^T \mathbf{c}^O + J_{t+1}^1(\mathbf{C}, \mathbf{b}_t^2, \mathbf{n}_t^2, \mathbf{y}_t^2 - \mathbf{s}_t, \boldsymbol{\varphi}_t^2)$$



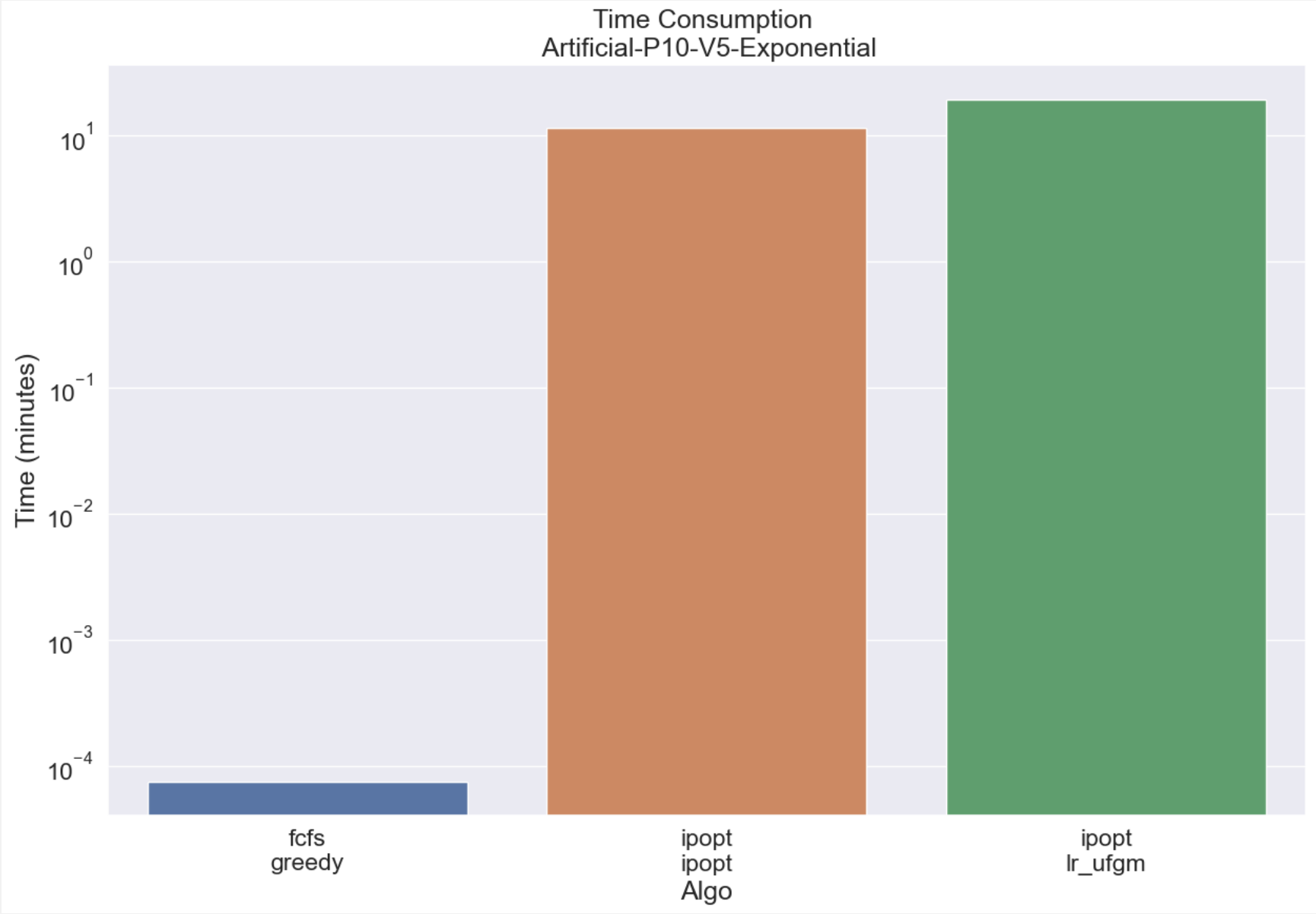


Sea Cargo Appendix. Decomposition Structure

Theorem. *Two-Level Relaxed Dynamic Program is separable in variables related to routes and service types (i.e. in* $(Q_{r,\sigma}, Z_{r,\sigma}, N_{r,\sigma}, F_{r,\sigma}, Q^i_{r,\sigma}, N^i_{r,\sigma}, A^i_{r,\sigma}, y^{HS}_{r,\sigma}, y^{OS}_{r,\sigma}, b_{t,r,\sigma}, n_{t,r,\sigma}, \varphi_{t,r,\sigma})$*), given that* λ_b, θ_b, $\lambda_{b,\sigma}$ *are fixed. Moreover, the following decomposition holds for* $t \geq 1$*:*

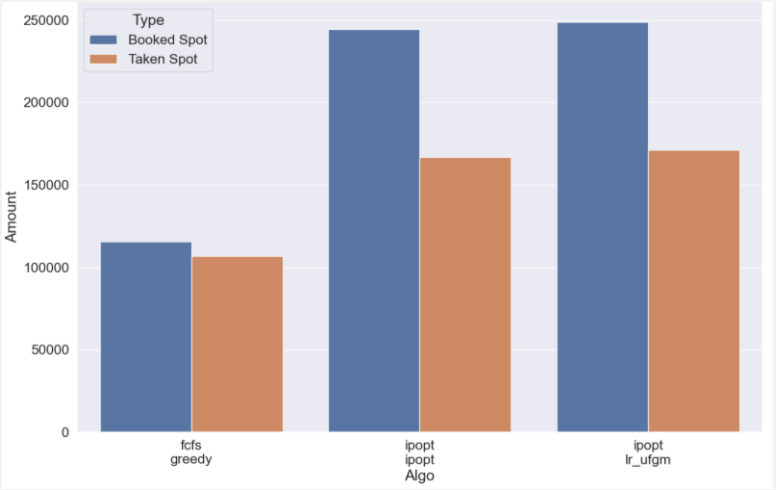
$$
\begin{aligned}
&KR_t^{1,\boldsymbol{\lambda},\boldsymbol{\theta},F,s,y^H}(\mathbf{C}, \mathbf{b}_t^1, \mathbf{n}_t^1, \mathbf{y}_t^1, \mathbf{l}_t^1, \boldsymbol{\varphi}_t^1) = \\
&\quad - \sum_{r:T_{start(r)}<t} \sum_{\sigma\in\Omega} \left((l^1_{t,r,\sigma} + n^1_{t,r,\sigma}) \sum_{b\in r} [\lambda_b \times \chi_\sigma + \theta_b \times \omega_\sigma + \lambda_{b,\sigma}] \right) \\
&\quad - \sum_{r:T_{strat(r)<t}} \sum_{\sigma\in\Omega} \left(w_\sigma \times (l^1_{t,r,\sigma} + n^1_{t,r,\sigma} - \varphi^1_{t,r,\sigma}) + w^e_\sigma \times \varphi^1_{t,r,\sigma} \right) \times \left(\sum_{b\in r} \theta_b \right) \\
&\quad + \sum_{r\in\mathcal{R}} \sum_{\sigma\in\Omega} v^{\boldsymbol{\lambda},\boldsymbol{\theta}}_{t,r,\sigma}(b_{r,\sigma}) + \sum_{b:\ travel} (\lambda_b V_b + \theta_b W_b + \sum_{\sigma\in\Omega} \lambda_{b,\sigma} V_{b,\sigma}) \\
&- \sum_{a=(g,t^*)\to(g',t'),T_a\geq t} \sum_{\sigma\in\Omega} c^H_{g,\sigma} y^H_{a,\sigma} - \sum_{T\geq t'\geq t} \mathbf{c}^{O\intercal}\mathbf{s}_{\mathbf{t}'} - \sum_{T\geq t'\geq t} d_{t'} \mathbf{y}^{1\intercal}_{t'+1} \mathbf{c}^S - \mathbf{c}^{O\intercal}\mathbf{y}^1_{T+1}
\end{aligned}
$$

Sea Cargo Appendix.
Artificial-P10-V5-Exponential: Execution Time



Sea Cargo Appendix.
Maersk-P4-V4-Linear: Containers and Profit

Spot Market: Booked vs Shipped
Maersk-P4-V4-Linear



Allotment Market: Booked vs Shipped
Maersk-P4-V4-Linear

