



Resort Revenue Management

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Paper 1 [Working Paper]: Theory Behind Randomized Linear Programming for Resort Revenue Management with Ancillary Values and Free Upgrade

Paper 2 [Work In Progress]: Data-Driven Resort Revenue Management



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Business Context

- Golf Resort: **hotel, golf, pub, bar, shop, restaurant**





- Regular Revenues (**certain**) + Ancillary Revenues (**uncertain**)
- Works from **May** to **October**
- Price categories are **fixed** in advance:

$$early = late \leq spring = fall \leq middle \leq peak$$

May → October

early	spring	middle	peak	middle	fall	late

Motivation

- A guest books a **single hotel night**
- Maximum price is **$3K**, minimum is **$250**
- Then spends **$5K** for a bottle of wine







- **Who are such clients?**
  The resort likes them!

- Opportunity: **10 guests**, **5 days** $\Rightarrow$ **$250K** within a **week**!

Prices: $CAD / $USD

Service Level	Peak	Mid	Spring/Fall	Early/Late
Cabot Double	$530/$417	$475/$374	$400/$315	$265/$209
Cabot King	$525/$413	$475/$374	$395/$311	$265/$209
Deluxe Double	$610/$480	$550/$433	$460/$362	$305/$240
Deluxe King	$615/$484	$555/$437	$460/$362	$310/$244
Deluxe King Balcony	$665/$524	$600/$472	$500/$394	$335/$264
2-Bedroom Villa	$1570/$1263	$1410/$1110	$1175/$925	$785/$618
4-Bedroom Villa	$3135/$2469	$2820/$2020	$2350/$1850	$1570/$1236
2-Bedroom Hillside Home	$1570/$1263	$1410/$1110	$1175/$925	$785/$618
3-Bedroom Hillside Home	$2350/$1850	$2115/$1665	$1765/$925	$1180/$929
4-Bedroom Cliffs Residence	$3135/$2469	$2820/$2220	$2350/$1850	$1570/$1236
5-Bedroom Cliffs Residence	$3920/$3087	$3525/$2776	$2940/$2315	$1965/$1547

- Several **service levels** (room categories)
- From 7 to 11, in different years (2016 – 2023)
- **Total Capacity**: >= 72

Case Study

1. Inputs: prices, capacities, individual ancillary spending

2. Output: RM system

3. Decision-Making Mechanism: smart personalized bid-pricing



Datasets:

- golf resort (confidential), 3 years
- resort & hotel (public), 3 years

Restrictions:

- free upgrades, no downgrades
- prices are fixed prior to the beginning of the season



Current Practice

- FCFS (first come, first served) – benchmark case
- **Pros**: easy to implement, intuitive
- **Cons**: no personalization, doesn't account for ancillary revenues, cancellations, no-shows
- **Opportunities**: ancillary profits, customer-centric view, personalized decisions, free upgrades





Randomized LP:

- **computationally** tractable
- induced bid prices are easy to **interpret**
- better than deterministic LP as it captures the **entire distributions of RVs**
- **easy to show** on sampled and historical trajectories

Resort RM: Practice. Prototype Overview

Descriptive Analytics

Predictive Analytics

Prescriptive Analytics

Data Engineering

Data Exploration

Ancillary Revenues

Uncensoring

Cancellations

Data Model + Sampling Opportunity

Operational Model





Resort RM - Theory: Opportunities and Contribution

Theory

Uncensoring



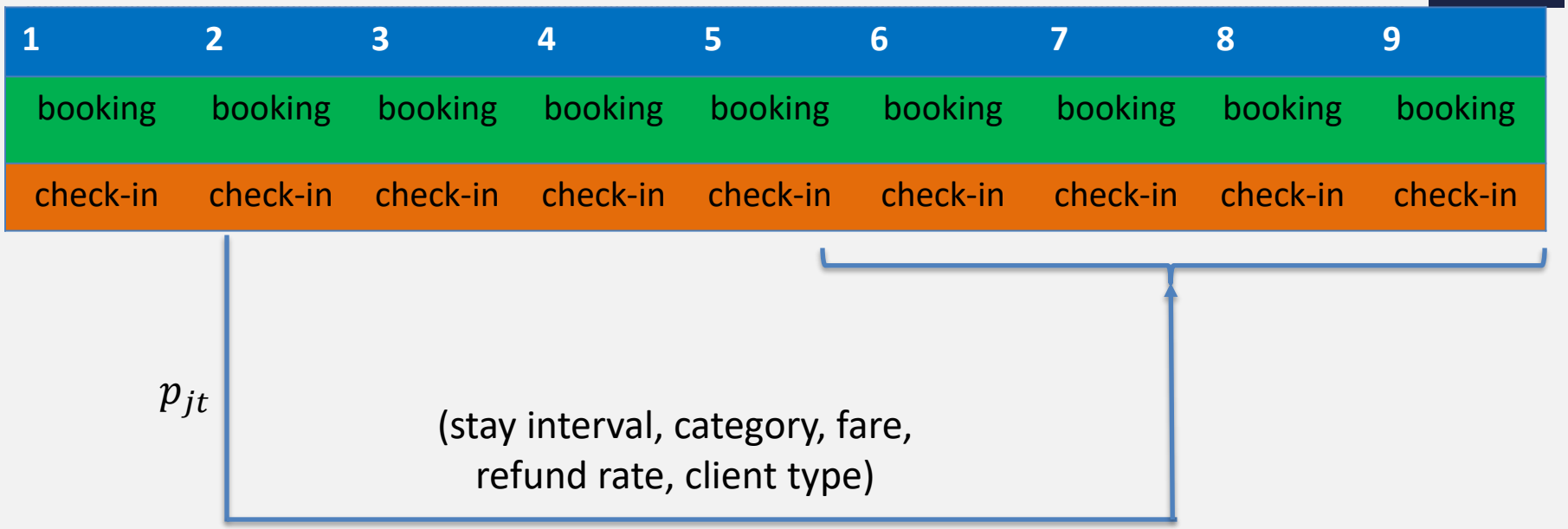
- GLM (generalized linear models)
- Parametric EM (expectation-maximization)

Operational Model

- RLP (randomized linear programming)
- Asymptotic optimality and inequalities



Operational model: setup



- Fixed number of **possible reservation request types**
- At **most one reservation** during each booking stage
- **Reservation Request** is a tuple of the form

  (stay interval, category, fare, refund rate, random ancillary value)
- Booking stage outcome is a binary random vector with probability p_{jt} to get **request j at time t** (1 in component j)
- **Ancillary Value** depends on the request type and time period

Operational model: client behavior and decisions

Client Behavior:

- On the check-in stage, each booking may result in one of the uncertain outcomes**: (1) Cancellation, (2) Show Up**
- Natural to introduce **overbooking** and **free upgrades**
- Upgrades and declines have **cost** (differentiated by booking request type)

Decision Variables:

- $\boldsymbol{y}$: booking acceptances (decisions at booking stages)
- $\boldsymbol{w}$: service class allocations (decisions at check-in stages)



Operational model: dynamic program

- $\boldsymbol{Value}(State)$ = optimal revenue with accepted requests $\boldsymbol{y}$ and allocations $\boldsymbol{w}$
- **Pre-Booking Value Function**:



- **Post-Booking Value Function:**



Reference: Sumit Kunnumkal, Kalyan Talluri, Huseyin Topaloglu (2012). **A randomized linear programming method for network revenue management with product-specific no-shows.** Transportation Science, 46 (1), 90-108

Modelling Approaches

- **Static Horizon:** all allocation decisions are made in the final time period (**airline RM**)
- **Rolling Horizon*:*** new customer reservations are accepted after other clients have arrived (**hotel RM**)
- **Sliding Horizon:** Same information as in rolling horizon, but with an opportunity to decide at the final time period (**military RM**)
- **Mixed Horizon**: intermediate between the static and rolling horizon

Family Of Methods

- **Deterministic LP**: uncertainties are replaced by expected values, and the objective is linear
- **Randomized LP**: multiple trajectories are available from history (or sampled); each of them is solved with perfect-information LP, and the resulting bid prices are averages over all trajectories





Family of Randomized LP

Closest to the referenced paper
Sumit, Talluri, Topaloglu - 2012

- ***Randomized LPs based on Rolling Horizon*****:** *One sampled trajectory with demands, shows-up/cancellations, and ancillary values*
- ***Randomized LPs based on Static Horizon****: Demand trajectory is sampled once, show-up/cancel trajectories are sampled several times, and ancillary values are set to their expectations*
- ***Randomized LPs based on Mixed Horizon***: *One sampled semi-trajectory with demands, shows-up/cancellations, and ancillary values are set to expectations*
- ***Partially Randomized LP:*** *Something is replaced with expectations, while the rest is sampled*

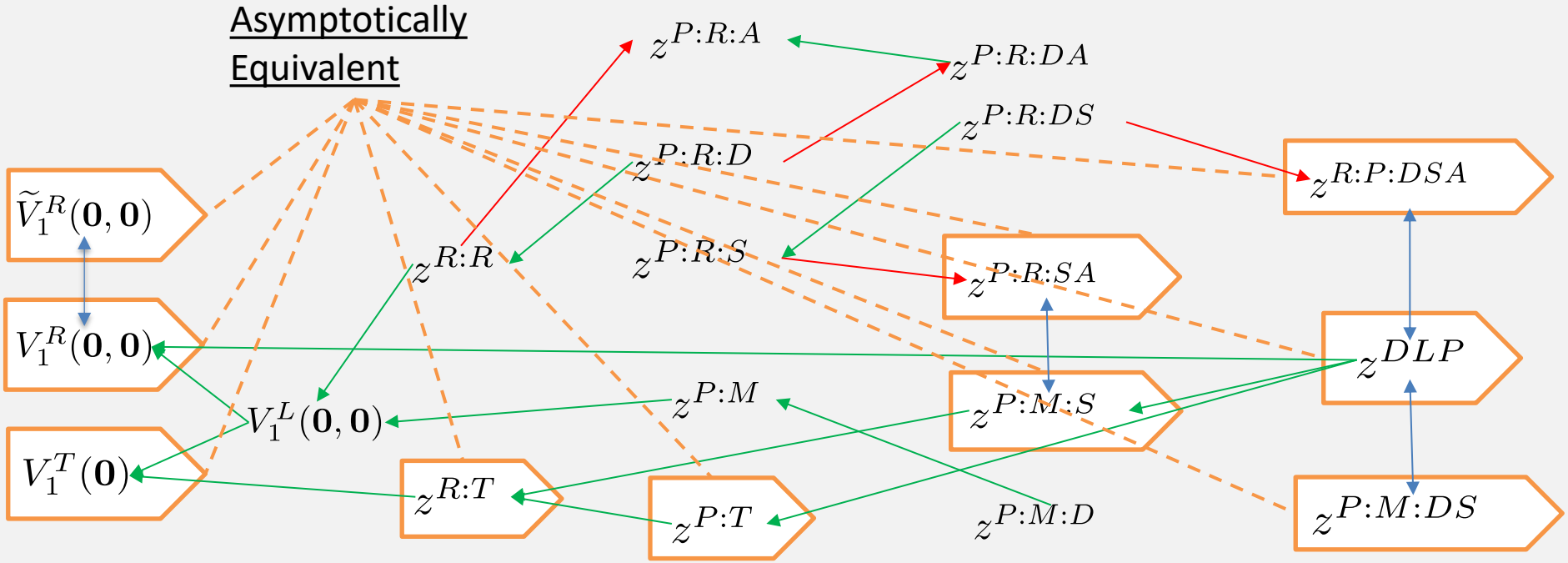






Summary of Theoretical Results

Theorem. The figure below depicts all the established inequalities together, where each arrow ← represents ≤ inequality



1. V-functions denote the pre-booking value functions of the optimal policy based on the optimal dynamic program
2. Subscripts in value functions describe the setting (R for rolling horizon, T for static horizon, L for sliding horizon)
3. R in the prefix states that the original Randomized LP form is applied, and P in the prefix abbreviates to Partially Randomized LP
4. R, T, M in the middle describe the configuration (Rolling Horizon, Static Horizon, Mixed Horizon)
5. For PRLP, the suffix describes what is replaced with the expected value. D in suffix means demands are expected, S means that show-ups/no-shows/cancellations are expected, A means that ancillary values are expected

Theoretical Results: Proof of Asymptotic Optimality

1. $Rolling\ Horizon\ DP \leq Deterministic\ LP$
2. $Rolling\ Horizon\ DP \sim Deterministic\ LP$ (using the coupling argument)
3. $Static\ Horizon\ DP \leq Randomized\ LP \leq Deterministic\ LP$
4. $Static\ Horizon\ DP \sim Deterministic\ LP$ (using the coupling argument)
5. $Randomized\ LP \sim Rolling\ Horizon\ DP$

Asymptotic Optimality holds only for a subset!

DP = Dynamic Program
LP = Linear Program



Resort RM: Theory. Empirical Study

Setting	Confidence Interval	Average Lift
Stylized	[1.294, 1.323]	1.308
Medium	[1.289, 1.301]	1.301
Golf-2018	[1.234, 1.238]	1.236
Golf-2022	[1.184, 1.188]	1.186

Most Effective In Practice:

1. R:T (majority of cases)
2. P:R:SA (sometimes)

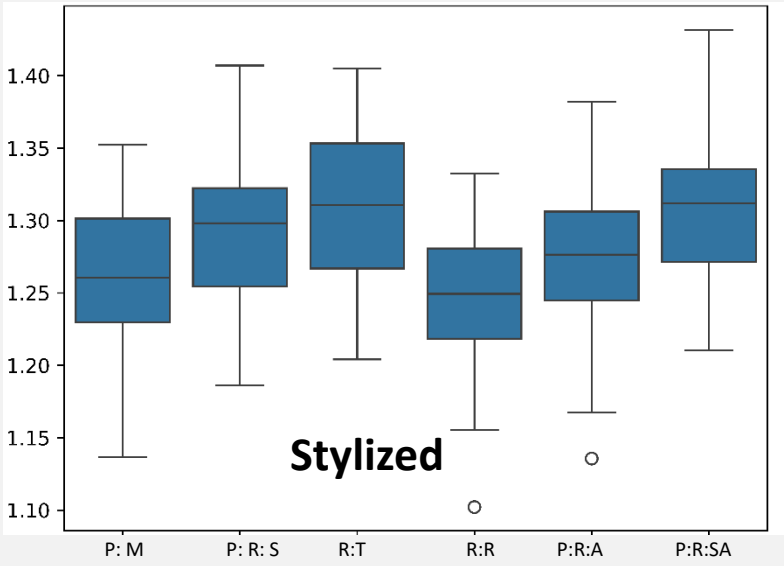
Important Factors:

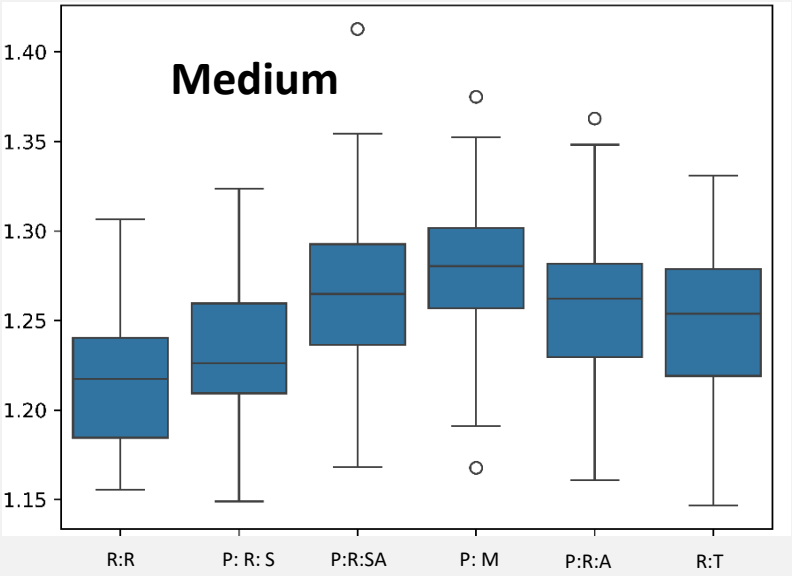
1. Client Differentiation
2. Generous Fraction
3. Ancillary/Room Revenue Ratio
4. Load Factor
5. Show Probability

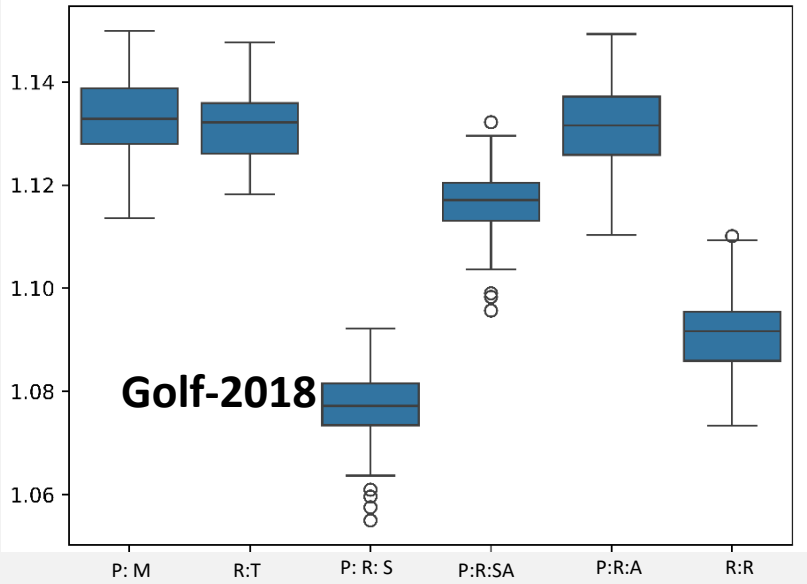


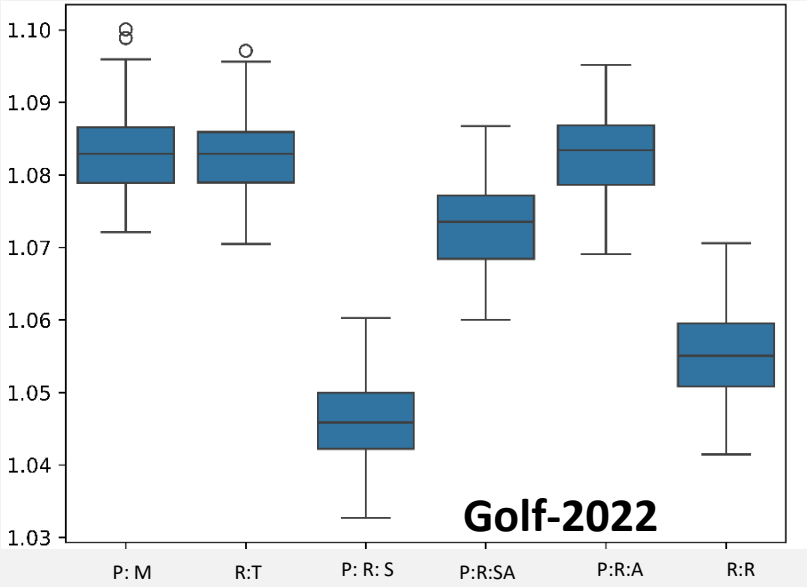


Resort RM: Theory. Ancillary Impact









Resort RM: Practice. Data-Driven Components

- **Ancillary Revenue**
  - Log-Model or Id-Model
  - ML Algorithms (RF, XGBoost, NN, Lasso, Ridge Regression, Adaboost, Gradient Boosting, etc.)
- **Demand Uncensoring** (Expectation-Maximization Algorithm, Generalized Linear Models)
- **Cancellations**
  - Probability of Cancellation
  - Duration to Cancellation
- **Decisions**
  - Randomized Linear Program
- **Data Source**
  - Private Resort in Canada
  - Public Data from Portugal



Resort RM: Practice. Results on Private Data - GLM

Ancillary Value Model	Ratio 2016	Ratio 2017	Ratio 2018
Gradient Boosting – Log	2.4%	1.9%	3.3%
Gradient Boosting - Id	3.2%	2.8%	4.0%
Random Forest - Log	2.5%	1.9%	3.4%
Random Forest - Id	3.2%	2.7%	4.0%
Ridge Regression – Log	2.4%	1.8%	3.2%
Ridge Regression - Id	3.0%	2.5%	3.5%
Extreme Gradient Boosting – Log	2.4%	1.9%	3.3%
Extreme Gradient Boosting - Id	3.2%	2.8%	4.0%
Adaptive Boosting – Log	2.3%	1.9%	3.2%
Adaptive Boosting - Id	3.3%	3.0%	3.9%
Neural Network – Log	2.3%	1.9%	3.3%
Neural Network - Id	3.0%	2.7%	3.7%
LASSO Regression – Log	2.1%	1.6%	3.1%
LASSO Regression - Id	2.9%	2.5%	3.6%

- Uncensoring performed with Generalized Linear Models
- Log at suffix tells that log-transformed targets were learned and predicted
- The ratio describes the performance of our approach relative to the FCFS (first come, first served) policy
- Optimistic performance is highlighted in yellow, and pessimistic is highlighted in red
- Decision with bid prices obtained by Randomized LP (P-R-SA)

Resort RM: Practice. Results on Private Data - EM

Intensity Stratification	Split Stratification	Intensity Form	Lift 2016	Lift 2017	Lift 2018
cmn-cmn	cmn-cmn	const id	3.5%	2.6%	3.6%
cmn-cmn	cmn-cmn	linear sigmoid	2.0%	2.1%	3.2%
cmn-cmn	cmn-season	const id	3.3%	3.1%	4.6%
cmn-cmn	cmn-season	linear sigmoid	3.4%	3.7%	4.1%
cmn-season	cmn-cmn	const id	3.8%	2.3%	2.7%
cmn-season	cmn-cmn	linear sigmoid	2.9%	3.1%	2.8%
cmn-season	cmn-season	const id	3.8%	2.9%	3.8%
cmn-season	cmn-season	linear sigmoid	2.7%	2.7%	3.3%
wkdy-cmn	cmn-cmn	const id	2.8%	3.9%	3.1%
wkdy-cmn	cmn-cmn	linear sigmoid	2.4%	2.3%	3.0%
wkdy-cmn	cmn-season	const id	2.5%	2.8%	4.1%
wkdy-cmn	cmn-season	linear sigmoid	3.6%	3.8%	3.7%
wkdy-season	cmn-cmn	const id	3.1%	3.3%	2.5%
wkdy-season	cmn-cmn	linear sigmoid	2.2%	3.3%	3.0%
wkdy-season	cmn-season	const id	2.4%	3.9%	3.1%
wkdy-season	cmn-season	linear sigmoid	3.7%	3.0%	3.1%

Hotel RM: Practice. Results on Semi-Real Data

Intensity Form	Year	Ratio Resort	Ratio City
Linear Sigmoid	2015	7.4%	1.2%
Constant	2015	9.7%	6.8%
Linear Sigmoid	2016	9.4%	4.9%
Constant	2016	14.5%	8.4%
Constant	2017	11.7%	6.1%
Linear Sigmoid	2017	7.5%	3.7%

EM uncensoring

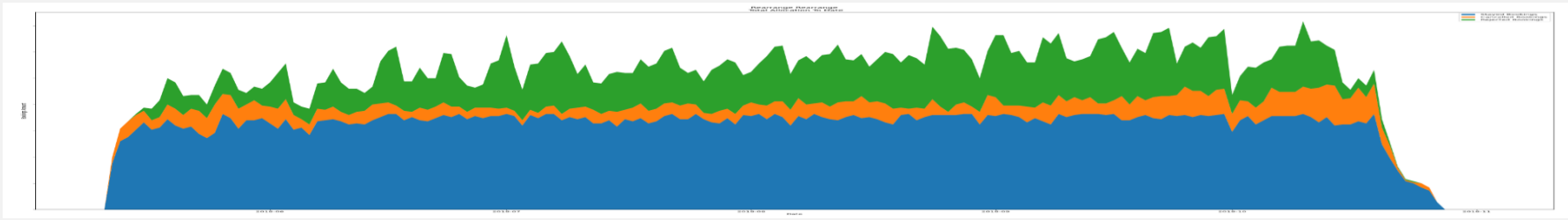
Year	Ancillary	Target	Ratio Resort	Ratio City
2015	Ridge Regression	Log	7.9%	4.5%
2016	Ridge Regression	Log	6.4%	4.7%
2017	Ridge Regression	Log	8.3%	5.9%

GLM uncensoring

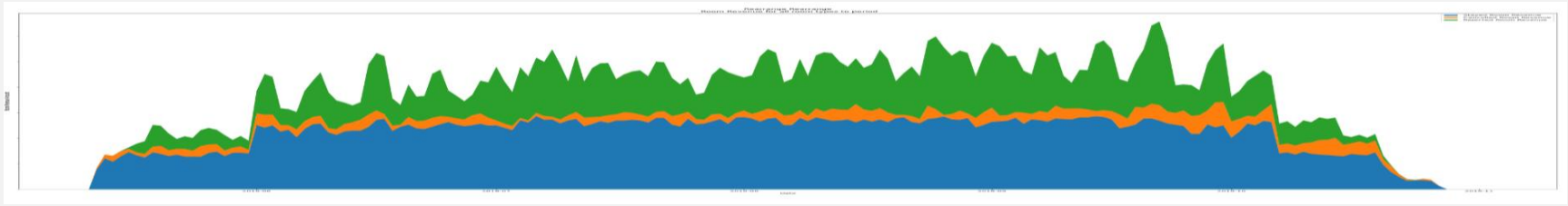
Semi-Real Data:

- Demand information is real
- Ancillary model is taken from private data (ridge regression, distorted by multiplying coefficients on random numbers)
- The ratio describes the performance of our approach relative to the FCFS (first come, first served) policy

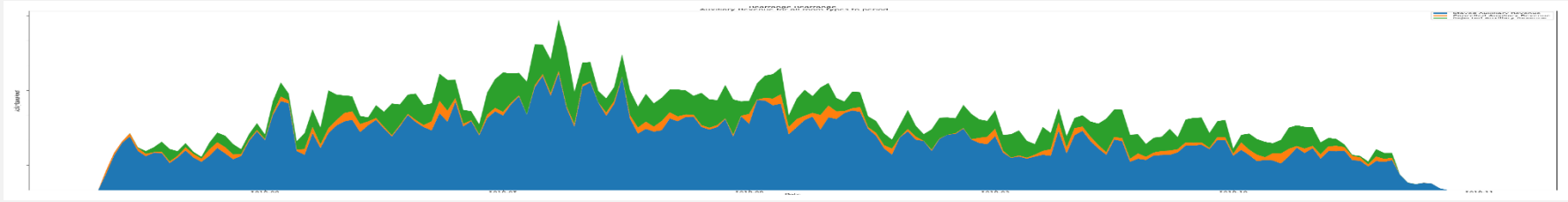
Hotel RM - Practice. Resort case, Parametric EM, Random Forest ancillary, Rolling Horizon RLP bid prices – Aggregate to Stay Date



Room Allocation



Room Revenue



Ancillary Revenue

- The upper shape in the graph represents the complete demand or revenue on the table correspondingly
- The flat, static blue shape represents reliable and stable capacity utilization for room allocation
- Majority of ancillary revenue is captured. Ancillary spending is highest during the peak seasons

Blue – Arrived
Orange – Cancelled
Green - Rejected

Resort RM: Selected literature

[1]. Sumit Kunnumkal, Kalyan Talluri, Huseyin Topaloglu. 2012. **A randomized linear programming method for network revenue management with product-specific no-shows.** Transportation Science, 46 (1), 90-108

[2]. Sheryl E. Kimes. 2000. **Revenue management on the links: applying yield management to the golf-course industry.** Cornell Hotel and Restaurant Administration Quarterly, 41 (1), 120-127

[3]. Kalyan Talluri, Garrett Van Ryzin. 1999. **A randomized linear programming method for computing network bid prices.** Transportation Science, 33 (2), 207-216

[4]. Sumit Kunnumkal. 2014. **Randomization approaches for network revenue management with customer choice behavior.** Production and Operations Management, 23 (9), 1617-1633

[5]. Garrett van Ryzin , Gustavo Vulcano (2017). **An expectation-maximization method to estimate a rank-based choice model of demand.** Operations Research 65(2):396–407

[6]. Guillermo Gallego, Richard Ratliff, Sergey Shebalov (2015). **A general attraction model and sales-based linear program for network revenue management under customer choice.** Operations Research 63(1):212–232

Resort RM: Summary and Contribution

Modelling:

- Free Upgrades
- Ancillary Values
- Rolling Horizon
- Overbooking
- Demand Uncensoring
- Cancellation Models



Practical Essay:

- Implementation from raw data to a profit-demonstrating project POC
- Unique Golf Revenue dataset







Theoretical Essay:

- Family Of Randomized LP Methods
- RLP Inequalities
- Asymptotic Optimality

Resort RM: Online Resources

- Implementation: https://github.com/smith-nekrald/hotel-theory.git
- Website: https://hotel-rlp-visual.onrender.com/
- Visualization: https://github.com/smith-nekrald/hotel-rlp-visual.git

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Thank you for your attention!

SSRN: https://papers.ssrn.com/sol3/papers.cfm?abstract_id=6126746